Humanoid Walking Control using the Capture Point

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Compliant Manipulation

Joint torque sensing & control for manipulation

Robustness: Passivity Based Control

Performance: Joint Torque Feedback (noncollocated)
Compliant Manipulation

Robustness: Passivity Based Control

Performance: Joint Torque Feedback (noncollocated)
Beyond Compliant Manipulation

Joint torque sensing & control for manipulation

DLR-Biped [Humanoids 2010]

Anthropomorphic Hand-Arm System
[Grebenstein, Albu-Schäffer et al, Humanoids 2010]

- Compliant actuation
- Antagonistic actuation for fingers
- Variable stiffness actuation in arm
- Robustness to shocks and impacts
From Manipulating to Walking

walking on arms
**Experimental biped walking machine [Humanoids 2010]**

- 6 DOF / leg

- \~50 kg

- Drive technology of the DLR arm

- Newly designed lower leg

- **Slim foot design:** < 10cm

- **Sensors:**
  - joint torque sensors
  - force/torque sensors in the feet
  - IMU in the trunk

- Developed within 10 month by student projects.

- **Allow for position controlled walking (ZMP) and joint torque control!**
First experiments with DLR-Biped

First experiment at Automatica Fair in 06-2010: ZMP preview control [Kajita, 2003]
Current approach: Walking control based on the Capture Point
[Englsberger, Ott, Roa, et al. IROS 2011]
Current Research Interests

Compliant Balancing Control
[Humanoids 2011, Tomorrow 9:50-10:10]

Walking Control

IRT Humanoid (U. Tokyo)
State of the art walking control for fully actuated robots

- Pattern Generator for desired CoM and ZMP motion
- ZMP based Stabilizer

E.g. Preview Control [Kajita, 2003]
Model Predictive Control [Wieber]
Goal: Simple, robust and flexible control framework for bipedal walking, which
- is not restricted to either position or torque control
- allows for higher level control strategies like push recovery and online step planning


Why to use Capture Point for walking?
- Simplifies planning (pattern generation)
- Extension to push recovery & online step adaptation
Assumptions

✓ one-mass model (robot modeled as point mass corresponding to the COM)

✓ COM at constant height $z_c$

✓ Base joint of pendulum is torque-free
Linear Inverted Pendulum (LIP)

\[
\ddot{x}_c = \omega^2(x_c - p)
\]

where \( \omega = \sqrt{g/z_c} \)

\[\dot{\sigma} = \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix} \sigma + \begin{bmatrix} 0 \\ -\omega^2 \end{bmatrix} p \]

with \( \sigma = [x_c \quad \dot{x}_c]^T \)

\[
\sigma(t) = \begin{bmatrix} \cosh(\omega t) & \frac{1}{\omega} \sinh(\omega t) \\ \omega \sinh(\omega t) & \cosh(\omega t) \end{bmatrix} \sigma_0 + \begin{bmatrix} 1 - \cosh(\omega t) \\ -\omega \sinh(\omega t) \end{bmatrix} p
\]
**Definition:**
The Capture Point (CP) is the point on the floor where the robot has to place the ZMP to come to a complete rest (=> COM over foot)

\[ x_c \bigg|_{t \to \infty} = p = x_{c,0} \cosh(\omega t) + \frac{\dot{x}_{c,0}}{\omega} \sinh(\omega t) + p - p \cosh(\omega t) \]

\[ p = x_{c,0} + \frac{\dot{x}_{c,0}}{\omega} \tanh(\omega t) \bigg|_{t \to \infty} \]

**Capture Point [1]**
\[ \xi = x_c + \frac{\dot{x}_c}{\omega} \]
\[ \omega = \sqrt{g/z_c} \]

[1] Pratt et al., 2006
Capture Point Dynamics

\[ \dot{\xi} = x_c + \frac{\dot{x}_c}{\omega} \]

\[ \ddot{x}_c = -\omega (x_c - \xi) \]

\[ \dot{\xi} = \dot{x}_c + \frac{\ddot{x}_c}{\omega} \]

\[ \dddot{x}_c = \omega^2 (x_c - p) \]

\[ \dot{\xi} = -\omega \dot{x}_c + \omega \xi + \omega \ddot{x}_c - \omega p \]

\[ \ddot{\xi} = \omega (\xi - p) \]
Derivation of Capture Point control (CPC)

\[ \dot{\xi} = \omega (\xi - p) \]
\[ \dot{x}_c = -\omega (x_c - \xi) \]

**Solution in time**
\[ \xi(t + dT) = e^{\omega dT} \xi(t) + (1 - e^{\omega dT}) p \]

motivates only stabilizing the unstable part of the dynamics

**Capture Point Control**
\[ p = \frac{\xi_d - e^{\omega dT} \xi}{1 - e^{\omega dT}} = \frac{1}{1 - b} \left( \xi_d - \frac{b}{1 - b} \xi \right) \]
Capture Point control (CPC)

Capture Point Control

\[ p = \frac{1}{1-b} \xi_d - \frac{b}{1-b} \xi \]

Closed loop dynamics

\[ \dot{\theta} = \begin{bmatrix} -\omega & \omega \\ 0 & \omega \frac{1}{1-b} \end{bmatrix} \theta + \begin{bmatrix} 0 \\ -\frac{0}{1-b} \end{bmatrix} \xi_d \]

with \( b = e^{\omega dT} \geq 1 \) \( \forall dT \geq 0 \) and \( \xi \leq \xi_c \)
Basic CP shift

- COM velocity always points towards CP
- ZMP "pushes away" the CP on a line
- COM follows CP
Foot-to-foot shift of CP

- predefined footprints
- desired end-of-step CPs at constant offset from foot centers
- desired ZMPs calculated according to CP control law
- only single support phase considered for planning, whereas control can handle double support
CP pattern generators

End-of-step reference (CPS)

- stepwise constant desired end-of-step Capture Points
- linear decreasing time until desired arrival

\[ p = \frac{\xi_d - e^{\omega \cdot dT} \xi}{1 - e^{\omega \cdot dT}} \]

Tracking reference (CPT)

- desired Capture Point follows a reference trajectory (ideal LIP) (shifted by \(dT\))
- time until desired arrival is held constant (design parameter)
Predefined footprints

Planning of end-of-step CPs

Scheduling for CPS or CPT control

CP control

support polygon projection

ZMP control

robot

\[ \xi = x_c + \frac{\dot{x}_c}{\omega} \]

\[ p = \frac{\xi_d - e^{\omega \cdot dT} \xi}{1 - e^{\omega \cdot dT}} \]
Projection of ZMP

- desired ZMPs projected to support polygon if needed
- tilting avoidance by explicit limitation of the ZMP to the support polygon
Simulations

CPT

CPS

Preview control

Preview control with dynamic filter

\[ x, x_d \]

\[ \zeta, \zeta_d \]
Experiments
Summary and Conclusions

✓ System structure: COM and CP have first order dynamics (COM follows CP (stable), ZMP “pushes” CP (unstable))

✓ CP control motivated by the solution in time of CP dynamics

✓ CP control stabilizes the unstable part of the dynamics

✓ Effective and simple tool for design of feedback controllers for bipedal walking robots
  => basis for push recovery and online step planning

✓ Robustness can be shown analytically, in simulation and experiments
✓ Push recovery / online step adjustment

✓ Consider projection of ZMP to support polygon in the controller design

✓ Extension of CP control to more general models (3D COM-motion ...)

Outlook
Thank you very much for your attention!

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