Research seminar Computational Arithmetic Geometry SS 2016

p-adic uniformization of Shimura curves

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Time: Fridays, 9 c.t. **Room:** SR 3

Consider a quaternion algebra B over a totally real field F, which splits at exactly one archimedean place τ and ramifies at a non-archimedean place $\mathfrak{p}$ (lying over a rational prime p, say). Let $\bar{B}$ denote the quaternion algebra obtained from B by changing the local invariants exactly at τ and $\mathfrak{p}$. Denote by G the algebraic group over $\mathbb{Q}$ defined by $B^\times$. Let $C = C^\mathfrak{p} \cdot C_\mathfrak{p}$ be a compact open subgroup of $G(\mathbb{A}_f)$, with $C_\mathfrak{p} \subset B_\mathfrak{p}^\times$ maximal. The associated Shimura curve has a canonical model $\mathcal{S}_C$ over F. The following theorem started the topic of p-adic uniformization of Shimura varieties:

Theorem [Cherednik '76]: *After extending scalars from F to $\bar{F}_\mathfrak{p}$ there is an isomorphism of algebraic curves*

$$\mathcal{S}_{C/\bar{F}_\mathfrak{p}} \cong \bar{B}^\times \backslash \left(\Omega^2_{F_\mathfrak{p}} \times G(\mathbb{A}_f)/C \right) \times_{F_\mathfrak{p}} \bar{F}_\mathfrak{p}.$$

Here, $\Omega^2_{F_\mathfrak{p}}$ denotes the rigid *Drinfeld upper halfplane*, $\mathbb{P}^1_{F_\mathfrak{p}} \backslash \mathbb{P}^1(F_\mathfrak{p})$, and the double-quotient on the right is understood as algebraic curve via GAGA.

For $F = \mathbb{Q}$ Drinfeld gave an integral version of this theorem, using a moduli interpretation that is only available in this special case. In higher-dimensional cases, whenever a moduli interpretation was possible, Rapoport and Zink [26] generalized this integral version.

The aim of this seminar is to understand all the objects involved here. More specifically, we want to shed some light on the preprint [5] that freely uses and extends many of these ideas to a *non-moduli* situation:

Theorem [Boutot-Zink '99]: *Let F, B, G and C be as above, the latter with arbitrary $\mathfrak{p}$-component. Furthermore, let $\breve{F}_\mathfrak{p}$ be the completion of $F_\mathfrak{p}^{\mathrm{ur}}$. Then there is an isomorphism of towers of formal schemes over $\breve{F}_\mathfrak{p}$,*

$$\bar{B}^\times \backslash \left(\hat{\Omega}^2_{F_\mathfrak{p}} \times_{\operatorname{Spf} \mathcal{O}_\mathfrak{p}} \operatorname{Spf} \breve{\mathcal{O}}_\mathfrak{p} \times G(\mathbb{A}_f)/C \right) \cong \mathcal{S}_C^\wedge \times_{\operatorname{Spf} \mathcal{O}_\mathfrak{p}} \operatorname{Spf} \breve{\mathcal{O}}_\mathfrak{p},$$

which is $G(\mathbb{A}_f)$- and Frobenius-equivariant.

We plan to have 4 talks on foundational material covering the different geometric settings (rigid and formal) and the group objects we want to parametrize later (finite flat and p-divisible groups). We will discuss Ω^d and its formal model in detail. After a report on Rapoport-Zink's work on moduli spaces, we will have 3 talks explaining the formalism of Shimura varieties, covering the basics, important examples and in particular Shimura curves. In the last 4 talks we turn to the preprint [5] and explain how it makes use of and generalizes Rapoport-Zink.

Note, that we don't meet on fridays after holidays (May 6 and May 27). If you are interested in giving a talk, please write an email to the organizers.

1 Talks

Talk 1 (Rigid analytic spaces and the example of Ω^d - **excess duration: 135 minutes**). Main references are [9, 21], for an overview see also [1, Parts 1,2],[32]

Start with an explanation why the naive viewpoint of locally analytic functions is not sufficient for non-Archimedean fields: E.g., give an example for a failure of the identity theorem. Introduce the main objects (and their basic properties) from [21] or [9]: Tate algebra and the Gauß norm, affinoid algebras, Noether normalization and Maximum principle ([21, Sections 1.2-1.4]). Continue with subdomains and admissible opens [9, Section 2.2] with an example (e.g. the Laurent domains of [9, Exercise 2.1.6]), introduce the Tate topology (and contrast this with the canonical topology, as in [9, Exercise 2.1.1]) where you can, if time permits, shortly mention how this is an example for a Grothendieck topology. State Tate's acyclicity theorem [21, Theorem 1.3.8] and its consequences for presheaves without proofs. Continue with rigid analytic spaces ([9, Definition 2.4.1]), the rigidification functor [21, Definition 1.6.9] and examples: $\mathbb{A}^{n,rig}$ ([9, Example 2.4.3]), $\mathbb{P}^{n,rig}$ ([9, Example 3.2.5] and [15, Example 4.3.3(5)]).

The aim of the last part is to discuss the example of Ω^d and the 'first proof' of [27, Proposition 1]. For this, first define the Bruhat-Tits building for GL_d with special emphasis on the case $d = 2$ [4, §1] (see also [25, Chapter 4] for an overview). Continue with the proof (as carefully as the remaining time permits) as sketched in [27, p. 50-51], filling the gaps using [13, §6].

Date: April 22, 2016 and April 28, 2016 (SR 11,2pm) **Speaker: Özge Ülkem**

Talk 2 (Formal schemes, Raynaud's formal models and $\hat{\Omega}^d$ - **excess duration: 135 minutes**). [2], [9, Section 3.3], but see also [31, 3].

Start with a short overview of adic rings (along [2, Section 7.1]), introducing the $\mathfrak{a}$-adic topology, adic rings, ideals of definition, separatedness and completeness. Continue with the definition of (affine) formal schemes as in [2, Section 7.1], including the formal completion [2, Example 7.1 (4)]. Treat the example 'For example [..] of $\mathbb{A}^n_K$' of [2, p. 161] in detail. Mention the characterization of formal schemes as sheaves on Nilp_Λ [31, Remark 2.1.7]. Continue with tfp and admissible formal schemes ([2, Definition 7.4 (1) and Remark 7.4 (2)]), explaining also the functor $X \mapsto X_{\mathrm{rig}}$ and the terminology of formal models [2, Proposition 7.4 (3) and Definition 7.4 (4)]. Contrast this with Raynaud's generic fiber functor [9, Exercise 3.3.8, Example 3.3.9 and Exercise 3.3.10]. In the remaining time say as much as possible about the (admissible) formal blow-up construction (a short summary is in [9, Exercise 3.3.11], but much more can be found in [2, Section 8.2]) and its consequences for the question of unicity of (Raynaud's) formal models.

Discuss in detail the example $\hat{\Omega}^d_F$ as a formal scheme and its relation to Ω^d_F, following [4]. Discuss the local moduli interpretation of Deligne in terms of free modules over complete separated $\mathcal{O}$-algebras [4, Section 4]. Mention the global moduli interpretation of Drinfeld [4, Section 5], but don't go too deep into details. Explain the correspondence with Ω^d via Raynaud's generic fiber [19, page 222].

Date: April 28, 2016 (SR 11,2pm) and April 29, 2016 **Speaker: Rudolph Perkins**

Talk 3 (Finite flat commutative group schemes). Main reference: [24, 29], but see also [28].

Recall/Introduce the main objects and their properties from [24, Lectures 1 and 2]: Affine group schemes and their characterisation in terms of Hopf algebras, the properties 'finite', 'flat', 'unramified', 'connected' and 'etale'. Cartier duality [28, Proposition 3.2.4], functor of points, isogenies and treat some examples from [29, §2] or [28, Section 3.1]. Particular attention should be payed to the problem of existence of kernels (which is easy, [29, (1.7)]) and cokernels [29, (1.8)]. Explain how the existence of cokernels [29, 3.4 Theorem] is established using the characterization of ffgs as sheaves for the fppf (faithfully flat finite presentation) site [16, Chapter 4] and [28, Section 5]. Also remark the result that the category of finite flat group schemes over a field is abelian, cf. [14, Proposition 6.5]. Prove the connected-etale exact sequence over a Henselian local ring [29, (3.7)]. Continue with Dieudonne theory for commutative ffgs of p-order over a perfect field of characteristic p: The result to be reached is [24, Theorem 28.3], but you can go along [20, Parts 1 and 2]: Define the Dieudonne ring and give the statements of [20, Theorem 1.2]. As time permits, discuss the example(s) from [20, Section 2.1 and 2.2].

Date: May 13, 2016 **Speaker: Juan Marcos Cerviño**

Talk 4 (p-divisible groups)**.** Main reference: [16]
Start with the definition of abelian varieties [16, (1.3) Definition] and examples (as elliptic curves [16, (1.7) Example] and [16, Examples (1.9) and (1.10)]).

Next, introduce isogenies along [16, Section 5, §1], including [16, (5.9) Proposition] and [16, (5.13) Corollary]. Define the isogeny category Isog and explain its main features (Poincare reducibility and semisimplicity of Isog) along [8, Section 7.6]. Next, introduce p-divisible groups as inductive systems of ffgs [16, (10.10) Definition] and explain the characterization as limits of fppf schemes in [16, (10.13)] (or [31, Section 1.1]). Also introduce Tate's formal Lie groups as in [30, (2.2)] and state the equivalence of categories as in Proposition 1 of ibid. Continue with the p-divisible group $X[p^\infty]$ attached to an abelian variety [16, (10.16) Definition] and [16, (10.17)]. Introduce polarizations [16, (11.6) Definition] and continue, if time permits, with the remainder of [16, Section 11, §1], i.e. with [16, (11.10) Proposition] and the explanations thereafter. Shortly explain the Rosati involution [16, Section 12, §3] (Section 12 can only be found on Moonen's website!). Continue with the extension of the Dieudonne-theory to p-divisible groups [17, III.5.6 Theoreme]. Mention that this gives rise to an association of Dieudonne module to abelian varieties, cf. [20, Section 3].

Date: May 20, 2016 **Speaker: Tommaso Centeleghe**

Talk 5 (Report on Rapoport-Zink's work)**.** The aim of this talk is to introduce certain moduli problems of p-divisible groups and report on representability results, following [26]. References are to this book, unless stated otherwise.

The main result of the first part is that the moduli $\mathcal{M}$ of quasi-isogenies to a given p-div.-gp. are representable, (2.16). To explain this, define quasi-isogenies of p-div.gp's, (2.8), and then sketch the idea of the proof (2.16) giving as much details as time permits:

- a crucial finiteness property in the building of p-adic GL_n (2.18)
- reformulate this as an approximation statement (2.27)
- cover $\mathcal{M}$ by subfunctors $\mathcal{M}_c$ bounding the *height*
- $\mathcal{M}_c$ is representable (2.28).

Finally, mention that under suitable conditions the quotient $\mathcal{M}/\Gamma$ exists, where Γ is a discrete subgroup of the quasi-isogenies of $\mathbb{X}$.

The second half of the talk introduces level structures, Weil descend, and the rigid coverings $\mathbb{M}$ of $\mathcal{M}$: More precisely, define the (p-adic) (PEL) data from (1.38)+(3.18) and explain how we get a p-div.gp. $\mathbb{X}_{/L}$ from this. State Def. (3.21), explaining the importance of Kottwitz' condition (iv), see also (3.58). For time reasons, you will have to be vague about *lattice chains*. We will think of them as 'level structure'. Anyway, we will apply theorem (3.25) only for trivial level structure. State (3.25) and its proof[1].

You should have 10 minutes left, to give the general definition of Weil descent (3.44-3.47).

Date: June 3, 2016 **Speaker: Konrad Fischer**

The next three talks introduce the machinery of Shimura varieties. There are many sources for this material. The original works by Deligne [10, 11] are the definite reference, but are not very gentle on the reader. We chose to follow the outline of Harris [18]. The speakers are welcome to supplement this (e.g. by [22] etc.).

Talk 6 (Sh.Var.I: Hodge structures, Deligne's axioms and Tori).
First a reminder on reductive groups: references are to [23]. Mention 'Chevalley's theorem' (10.25), linear alg. gp's (LAG):=finity type+affine=closed in some GL_n (4.8). Now restrict to LAG: smooth in char 0 (3.38), quotients exist! (5.21). Give def. of connected+simply connected (and what does that mean for $\mathbb{C}$). State: G° normal, $\pi_0(G)$ is a finite group (§5i). Give equivalent def's of unipotent, define $R_u(G)$ and give examples $(\mathrm{GL}_n, \mathrm{SL}_n, \mathrm{Sp}_{2n}, U_n, B_n, \mathbb{G}_a^n, \alpha_p)$. Define 'reductive':='$R_u(G_{\bar{k}}) = 0$' and sketch: $G_\mathbb{R}$ reductive $\iff$ $\exists$ Cartan involution.
Remind us of *tori*; particularly $\mathbb{S} := \mathrm{Res}_{\mathbb{C}/\mathbb{R}}(\mathbb{G}_m)$ and U_1. Show: $\mathbb{S}$ is affine. Give the s.e.s. $1 \to \mathbb{S}^1 \hookrightarrow \mathbb{S} \overset{N}{\to} \mathbb{G}_{m/\mathbb{R}} \to 1$ and $1 \to \mathbb{G}_{m/\mathbb{R}} \overset{w}{\to} \mathbb{S} \to U_1 \to 1$. Recall the representation theory of these groups, cf. [22, p.19+26+50].
Recall the exact sequences from [10, §1.1]. Define *Shimura data* (G, X) for reductive $G_{/\mathbb{Q}}$. Give some insight/motivation for the 3 'main' axioms (cf. [22, pp.30,54], [11, 1.1.14]). Define the system $M_K(G, X)$ and the $G(\mathbb{A}^f)$-action. Mention that for small $K \subseteq G(\mathbb{A}^f)$, approximation + Baily-Borel give algebraic structure over $\mathbb{C}$ (!). Explain 'connected components = 0-dimensional Shim.var.' by going through §2 of [10] or §5 of [22], assuming G^{der} is simply connected.

Date: June 10, 2016 **Speaker: Andreas Maurischat**

Talk 7 (Sh. Var. II: Symplectic groups, canonical models).
In [10, §5] Delinge constructs from a monomorphism of Shimura data (e.g. PEL $\hookrightarrow$ symplectic) an inclusion of Shimura varieties and gets canonical models for the PEL-case. Aim of this talk is to give the symplectic picture, define canonical models and say something on their construction in the symplectic case. References are to [22].

Begin by introducing the Shimura datum to a symplectic space, pp.66-67, and checking (SV1)-(SV6). Say something about the moduli interpretation (Prop. 6.3) (in terms of Hodge structures or, equivalently by Riemann's theorem, in terms of complex abelian varieties, Thm. 6.11).

Define the reflex field $E(G, X)$ of a general Shimura datum, 12.2-12.4(b). Then continue with special points, pairs and the homomorphism r_x, 12.5-12.7. Finally give the definition of a *canonical model*, 12.8. Maybe say something, how this compares to the definition 3.13 of [10]. Using the theorem of Shimura-Taniyama, Delinge in [10, 4.21] constructs a canonical model for the symplectic case. Much more details can be found in [22, §14]. Assuming Deligne's result ('there are many special points', [10, 5.1]), uniqueness is an easy consequence.

Date: June 17, 2016 **Speaker: Johannes Anschütz**

[1] 2 x 'clearly' + 2 x 'obviously' = proof

Talk 8 (Sh. Var. III: PEL-case, Shimura curves and 'strange models')**.**
References to [22]: Recall the structure of semi-simple k-algebras with involution $(B,\star)$ from [22, 8.3], for $k=\bar{k}$. Classify symplectic $(B,\star)$-modules and define the associated reductive groups for cases (A),(C); cf. [22, 8.7]. If B carries a *positive* involution there is a unique Shimura datum satisfying (1)-(4), Prop. 8.14, and it has a moduli interpretation, Thm. 8.17. Explain how to get a canonical model by 'embedding', cf. [22, p.115] and in general [10, 5.7].

What goes wrong in the Shimura curve case: $E(G,X)$ [22, 12.4(d)], and then §6 of [10], with details explained in [6].

Date: June 24, 2016 **Speaker: Konrad Fischer**

The remaining talks are on the preprint [5]. Note that we **don't** cover the construction of $\det_{\mathbb{N}}$ in §2 of the paper.

Talk 9 (Unitary Shimura curves as moduli after Boutot-Zink, part 1)**.**
Begin by introducing the general notation of [5], which will be used throughout the remainder of the seminar. The relevant objects ($F\subset K$ maximal totally real subfield in a CM-field, the division algebra B, the alternating nondegenerate bilinear form ψ, the unitary group $\tilde{G}^\bullet$, the compact open subgroup (level) $C\subset\tilde{G}^\bullet(\mathbb{A}_{F,f})$, ...) in [5, pp. 5–7] should be explained carefully. Emphasize that we assume that the level C is maximal at $\mathfrak{q}$ (cf. [5, p. 7]). This assumption will be dropped in Talk 11.

Continue by introducing the category $\underline{AV}$ of abelian O_K-schemes up to isogeny of order prime to $\mathfrak{p}$, the field E and the (PEL-)moduli problem $\underline{\mathcal{A}}_C$ on the category of $\mathcal{O}_E$-schemes. State the representability result for the sheafification $\mathcal{A}_C$ of $\underline{\mathcal{A}}_C$ for sufficiently small C ([5, Proposition 1.1]). If time permits, the proof could be sketched (shortly!). The $\mathcal{O}_E$-schemes $\mathcal{A}_C$ form a projective system for varying C. Explain the action of $G^\bullet(\mathbb{A}_f)$ on this projective system and why the projective limit exists as a scheme, use this to define the projective $\mathcal{O}_E$-scheme $\mathcal{A}_C$ for general C ([5, pp. 12–14]).

The main aim of this talk is to proof [5, Lemma 1.8], which states that over E_{p^∞}, $\mathcal{A}_C$ is isomorphic to the unitary Shimura curve Sh_C associated to $G^\bullet$ and (W,ψ). Skip [5, pp. 15–20] which will be discussed in the next talk. Start with the definition of Sh_C in [5, p. 21] and explain why, for sufficiently small C, Sh_C is a fine moduli scheme of a functor closely related to $\underline{\mathcal{A}}_C$. Then continue with the proof of [5, Lemma 1.8].

Date: July 1, 2016 **Speaker: Mirko Rösner**

Talk 10 (Uniformization of unitary Shimura curves after Boutot-Zink, part 2)**.**
Building on the previous talk, the main aim of this talk is to prove an integral uniformization theorem for unitary Shimura curves Sh_C with C maximal at $\mathfrak{q}$ ([5, Theorem 1.12]). Begin by defining the moduli problem $\breve{\mathcal{M}}$ of p-divisible $\mathcal{O}_{B_\mathfrak{p}}$-modules ([5, Definition 1.4]). By [26, Theorem 3.25] this functor is representable by a formal scheme locally formally of finite type over $\mathrm{Spf}\,\mathcal{O}_{\breve{E}}$. Continue by introducing the Weil descent datum on $\breve{\mathcal{M}}$ and define the action of $G^\bullet(\mathbb{A}_f)$ ([5, pp. 16-17]).

The key ingredient to prove the uniformization theorem is the uniformization morphism $\Theta:\breve{\mathcal{M}}\times\tilde{G}^\bullet(\mathbb{A}_{F,f}^{\mathfrak{p}})/C^{\mathfrak{p}}\to\mathcal{A}_C\times_{\mathcal{O}_E}\mathrm{Spec}\,\mathcal{O}_{\breve{E}}$. Define this morphism and study its fibers as well as compatibility with the Weil descent data on both sides ([5, pp. 17–20]). Explain why Θ induces an isomorphism of formal schemes following [26, 6.30].

Putting this together with the results of the previous talk that relate $\mathcal{A}_C$ and Sh_C, we are able to prove a first uniformization theorem for unitary Shimura curves ([5, Proposition 1.9]. For this, we need to introduce the notation on [5, pp. 24–25]. Conclude this talk by defining the moduli problem $\breve{\mathcal{N}}$ ([5, Definition 1.10]), which is closely related to $\breve{\mathcal{M}}$ ([5, Proposition 1.11]) and reformulate the uniformization theorem in the form [5, Theorem 1.12]. In the next talk we will see how $\breve{\mathcal{N}}$ is related to the formal scheme $\hat{\Omega}^d_E$ (cf. Talk 2).

Date: July 8, 2016 **Speaker: David-A. Guiraud**

Talk 11 (Unitary Shimura curves with bad level structure after Boutot-Zink)**.**
In order to get a better understanding for the results of the previous talk, we want to relate $\breve{\mathcal{N}}$ to the formal scheme $\hat{\Omega}^d_E$. Recall (or introduce) the global moduli interpretation of $\hat{\Omega}^d_E$ of Drinfeld for general d (cf. Talk 2 or [12, Section 2]), which can be used to relate $\breve{\mathcal{N}}$ and $\hat{\Omega}^d_E$ and to reformulate the uniformization theorem of the previous talk ([5, Theorem 1.12]) in terms of $\hat{\Omega}^d_E$ ([5, pp. 28–29]).

The main aim of this talk is to extend the results of the previous talk to unitary Shimura curves with bad level structure C, i.e. where C is no longer maximal at $\mathfrak{q}$. In this setting, the uniformization isomorphism will no longer be integral and we have to work in the category of rigid analytic spaces over $\breve{E}$. Begin by introducing the rigid analytic pro-space $\mathbb{M}$ associated to the functor $\breve{\mathcal{M}}$ and the étale covering map $\mathbb{M} \to \breve{\mathcal{M}}^{rig}$ ([5, pp. 29–30]). Continue with the definition of the functor $\underline{\mathbf{A}}_C$, that turns out to be the general fibre of the functor $\underline{\mathcal{A}}_C$ for maximal C and state the representability result for the sheafification $\mathbf{A}_C$. Use the uniformization isomorphism of the previous talk to deduce a rigid version of the uniformization isomorphism for general C ([5, pp. 30–31]).

Similarly to the proof of [5, Proposition 1.9], we can use the relation between $\mathbf{A}_C$ and the rigidification of the unitary Shimura curve to deduce [5, Proposition 1.13]. Again, we conclude this talk with reformulating the uniformization theorem in terms the rigid pro-analytic covering space $\mathbb{N}$ over $\breve{\mathcal{N}}^{rig}$ ([5, Theorem 1.15]). If time permits, it would be nice to shed more light on these covering spaces in terms of $\hat{\Omega}^d_E$ ([12, Section 3], see also the (more detailed) english translation of [4]). The statement of [5, Corollary 1.16] can be skipped.

Date: July 22, 2016 **Speaker: Gebhard Böckle**

Talk 12 (Uniformization of Shimura curves after Boutot-Zink)**.**
In this talk, we prove the main result of [5], the uniformization of Shimura curves as mentioned in the introduction (in particular in a non-moduli situation). Begin with the general setup in [5, pp. 45–46] and state the main results we want to prove ([5, Theorem 3.1] and [5, Corollary 3.2]). Since we did not introduce the map $\det_{\mathbb{N}}$, the second statement in [5, Theorem 3.1] can be omitted.

The main idea in proving [5, Theorem 3.1] is to embed the Shimura curve into a unitary Shimura curve of the type considered in Talk 9–11 and to apply our uniformization results for these Shimura curves ([5, Theorem 1.15]). In order to deduce the theorem, we need to know more about the connected components of the rigid analytic covering spaces. Begin with the construction of the division algebra B and the unitary group $G^\bullet$ ([5, pp. 47-49]). The associated unitary Shimura curve admits a p-adic uniformization by [5, Theorem 1.15] (in order for the notation to work, C is denoted by $C^\bullet$ now). Explain how to choose $C^\bullet$ in dependence of C and state the (crucial!) result that our Shimura curve is an open and closed subvariety of the contructed unitary Shimura curve following [10]. Then use the abstract properties of $\det_{\mathbb{N}}$ to deduce the uniformization theorem ([5, pp. 50–52]). For this, the action of $I^\bullet(\mathbb{Q})$ has to be made more explicit as in [5, Lemma 3.3]. The statement of [5, Corollary 3.4] can be skipped.

Date: July 29, 2016 **Speaker: Peter Gräf**

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