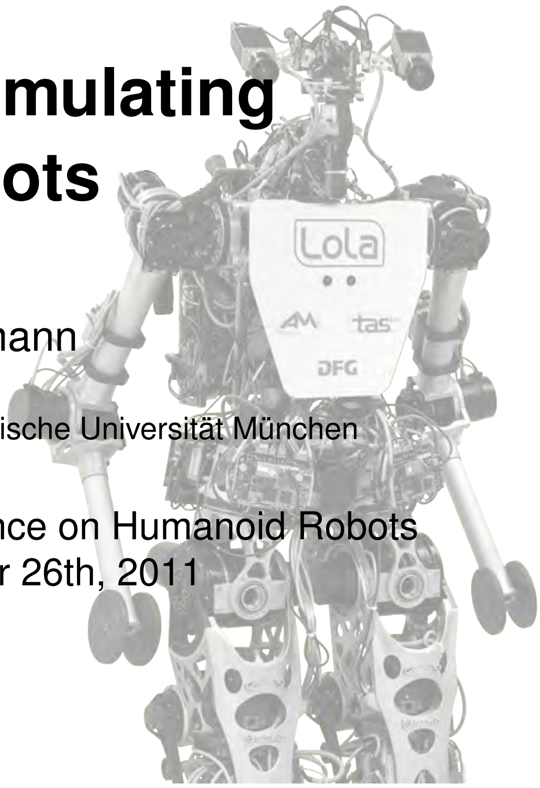
# Modeling and Simulating Biped Robots





Thomas Buschmann

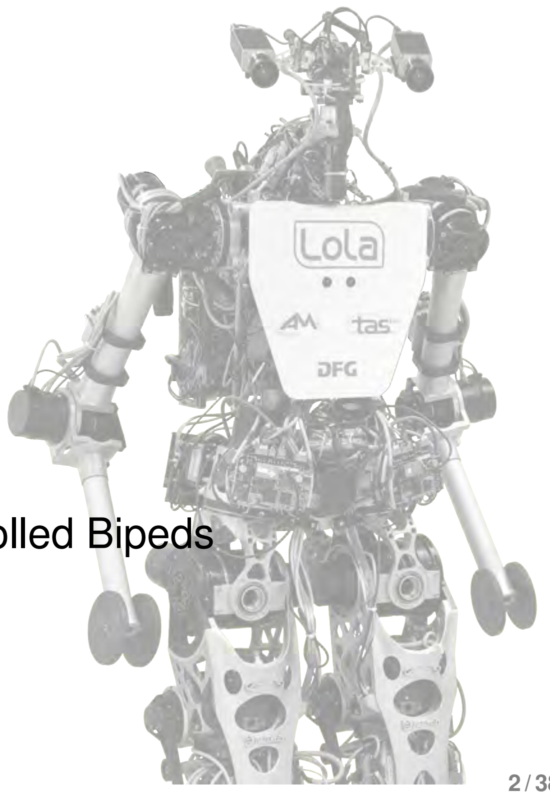
Lehrstuhl für Angewandte Mechanik, Technische Universität München



11th IEEE-RAS International Conference on Humanoid Robots
in Bled, Slovenia, October 26th, 2011

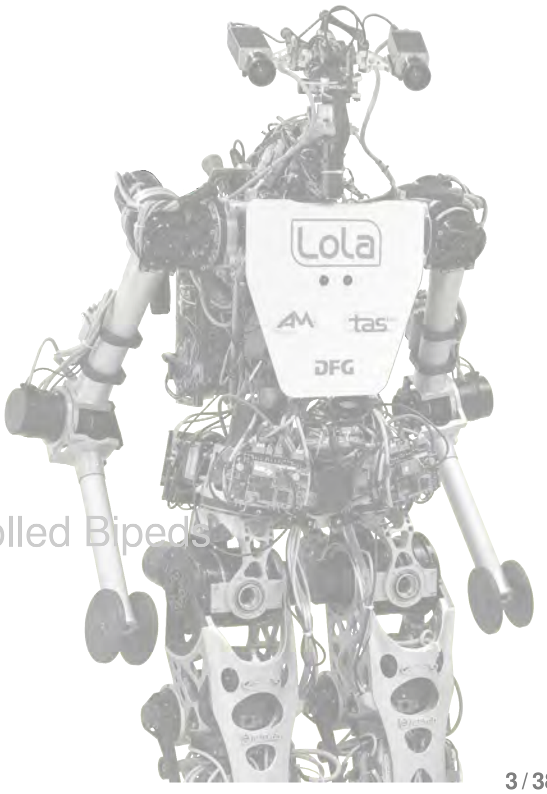



# Outline



# Outline





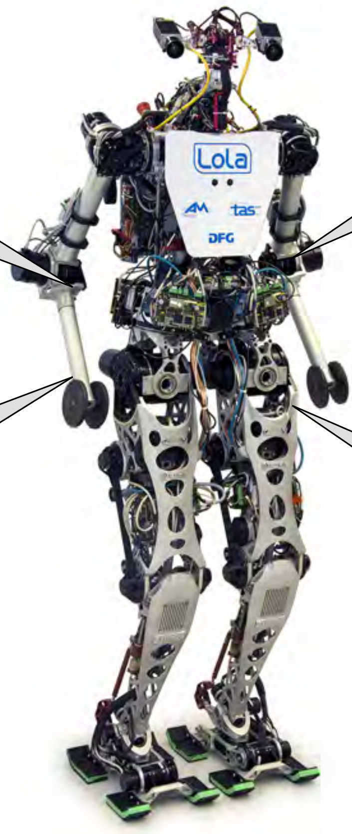





# System Overview – Lola

**Sensors**
- Encoders
- IMU
- FTS
- Cameras

**Drives**
- Electric motors
- Harmonic Drives
- Roller screw drives



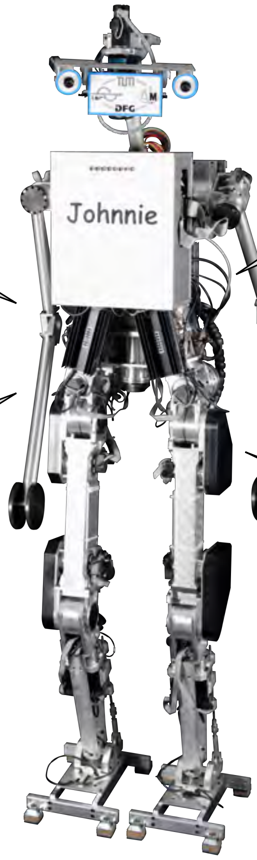

**Mechanical Design**
- Rigid links
- Compliant contact
- Drive kinematics
- Mass distribution

**Electronics**
- On-board PC
- Local controllers
- Sercos-III bus

# System Overview – Johnnie

**Sensors**
- Encoders
- IMU
- FTS
- Cameras

**Drives**
- Electric motors
- Harmonic Drives
- Ballscrew drives




**Mechanical Design**
- Rigid links
- Compliant contact
- Drive kinematics
- Mass distribution

**Electronics**
- On-board PC
- Centralized system



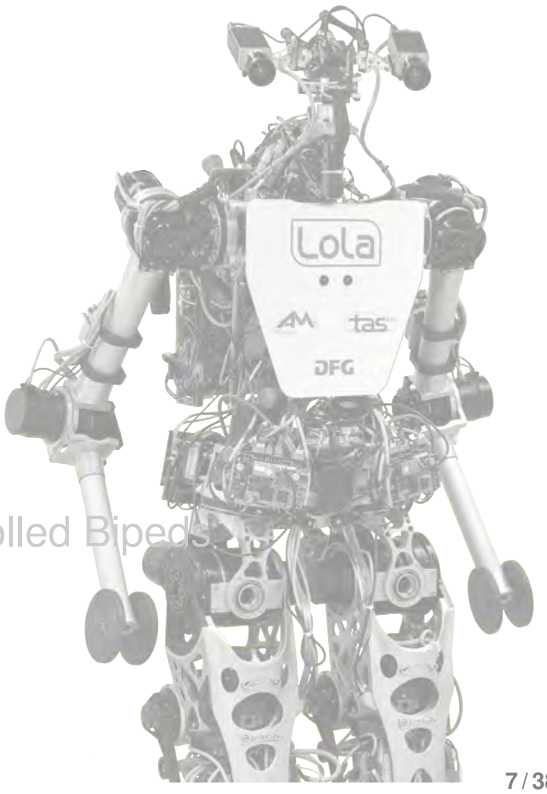


# Why Simulate?

- Prototyping
- Hardware development and sizing
- Safe testing environment
- Analyze robot dynamics
- Offline optimization

# Outline









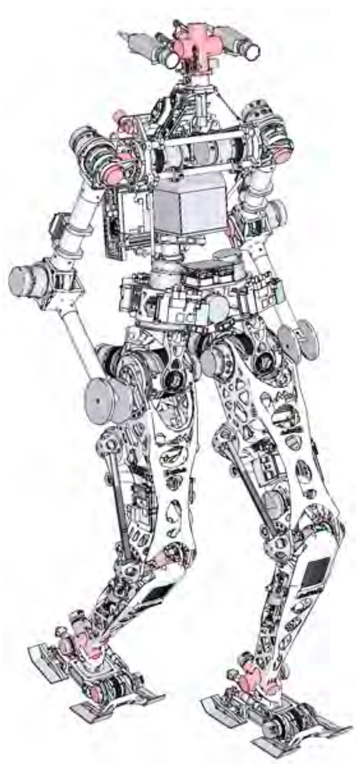

# Robot Model Components

**Rigid Body Mechanics**

- Rigid multibody dynamics
- Nonlinear drive kinematics

**Contacts**

- Unilateral contact with friction
- Compliance and damping



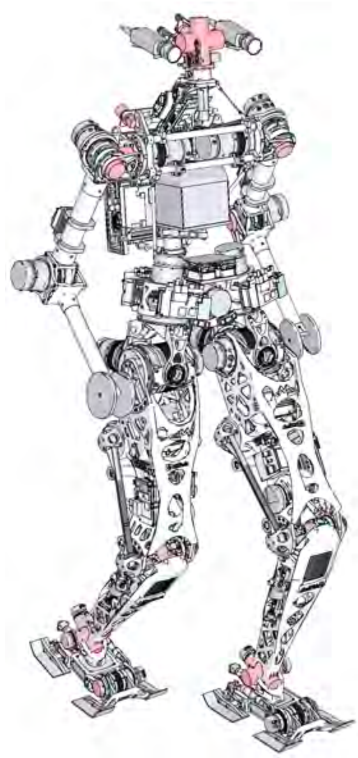
# Robot Model Components

**Drives**

- Electrical motor dynamics
- Gears: friction, elasticity?

**Sensors**

- Encoders
- FTS, IMU, . . .







# Multibody Dynamics

**Multibody Formalism Based on Newton-Euler-Jourdain**

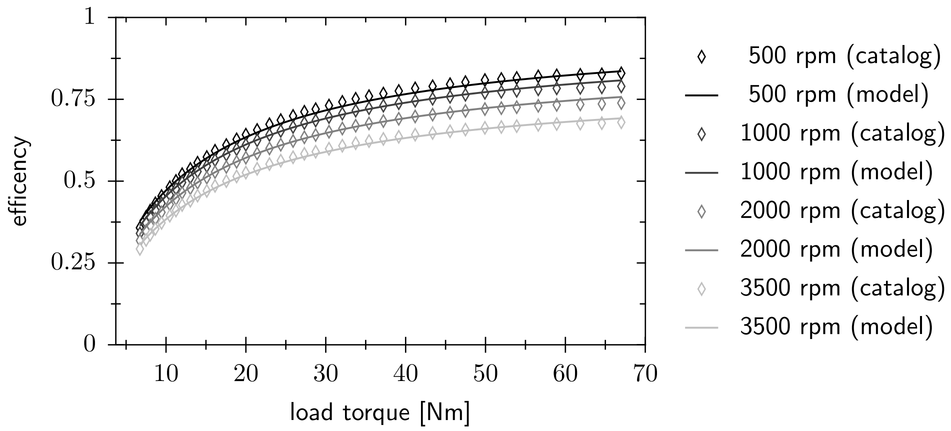
$$\sum_{i=0}^{N_{\text{bodies}}} \left\{ \begin{pmatrix} J_{T,o,i} \\ J_{R,i} \end{pmatrix}^T \begin{pmatrix} \dot{p}_i - F^i \\ \dot{L}_{O_i,i} + m_i \widetilde{r}_{O_i C_i} \ddot{r}_{O_i} - M^i_{O_i} \end{pmatrix} \right\} = \mathbf{0}$$

$$\Rightarrow \quad M\ddot{q} + h = Q$$

**Recursive Calculation**

- Recursive calculation of terms in EoM ($J_R$, etc.)
- Expoits “tree structure” (“holes” in jacobians, etc.)

# Gear Friction



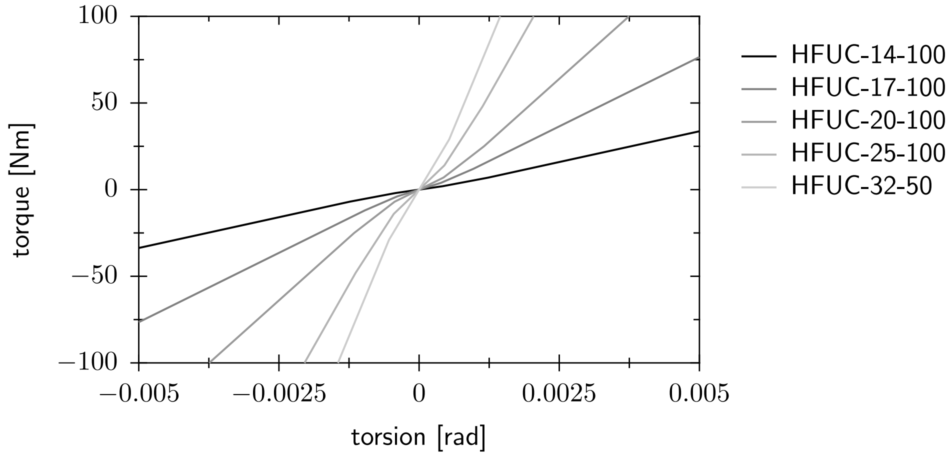

Efficiency for HFUC 25-100 gear.

$$T_f = -\text{sign}(\omega)(T_{f,0} + \mu \|T_l\|) - (b + \gamma \|T_l\|)\omega$$



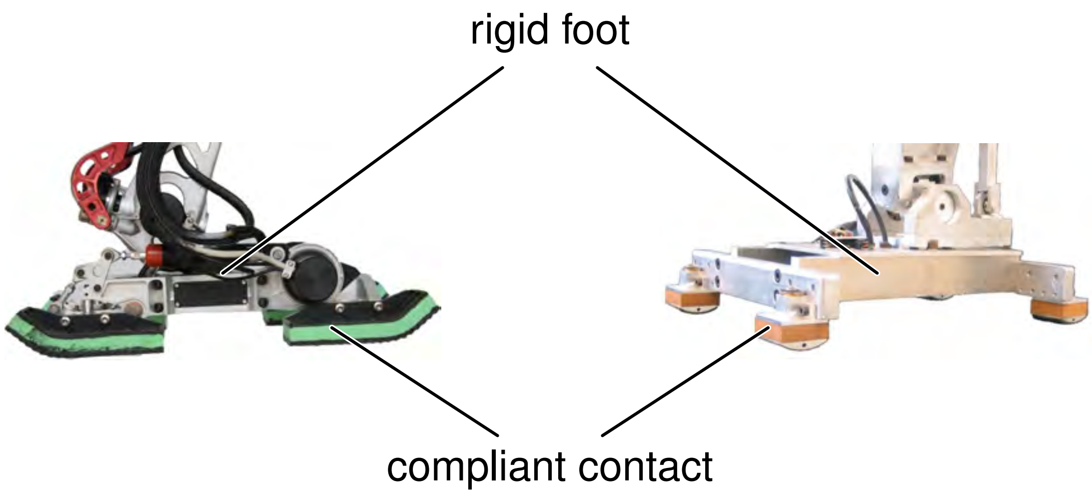


# Gear Elasticity




$$T_{\text{gear,elast}} = \begin{cases} K_0 \Delta\varphi & \text{for} \quad T_{\text{gear,elast}} < T_1 \\ T_1 + K_1 \Delta\varphi & \text{for} \quad T_1 \leqslant T_{\text{gear,elast}} < T_2 \\ T_2 + K_2 \Delta\varphi & \text{for} \quad T_{\text{gear,elast}} \geqslant T_2 \end{cases}$$

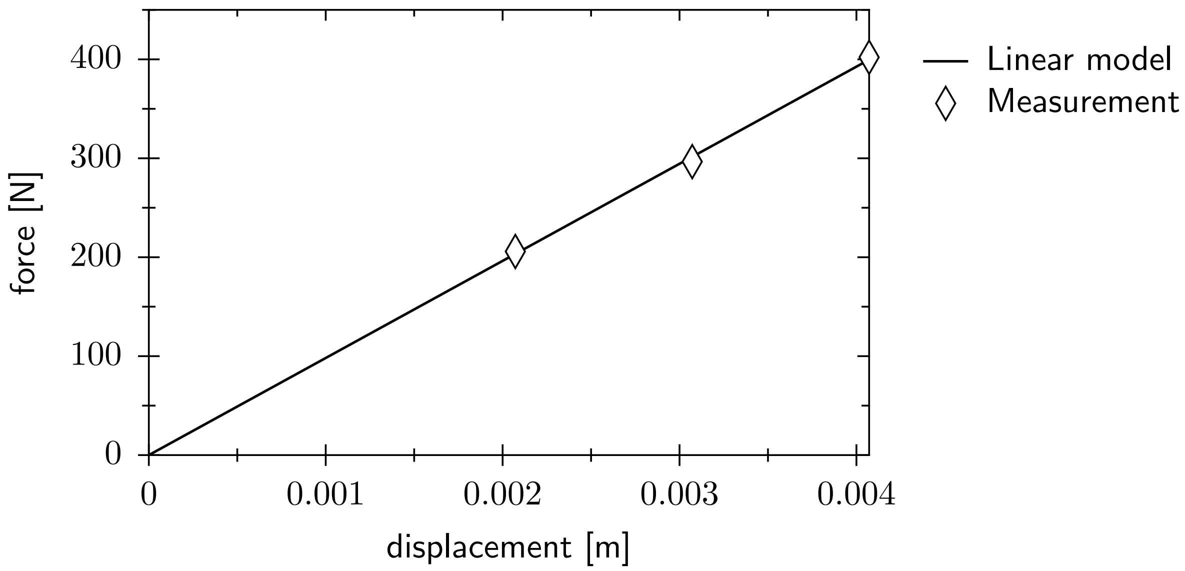
# Robotic Feet





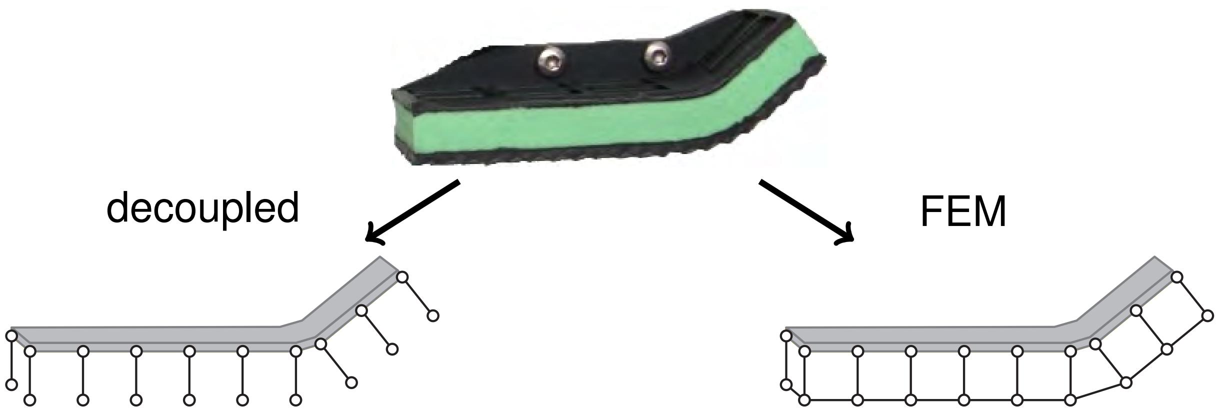



# Modelling – Stiffness




Quasi-static force-displacement relationship for Lola.

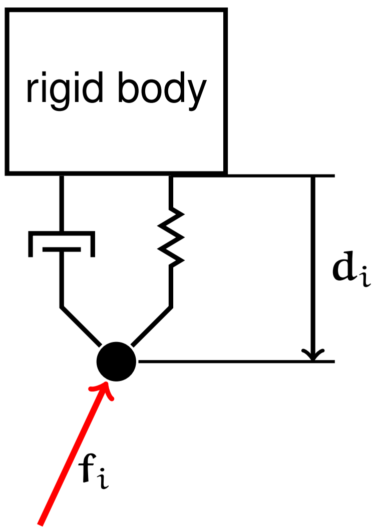
# Modelling – Geometry





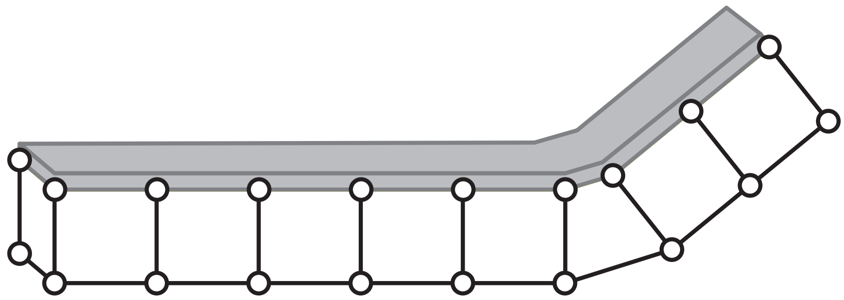



# Simple Spring-Damper Model




- $N$ decoupled spring-damper elements
- For every element: $f_i = C_i d_i + B_i \dot{d}_i$
- Deformation in body-fixed frame
- $D, C$ diagonal

# Finite Element Model



$$\mathbf{f} = \mathbf{B}\dot{\mathbf{d}} + \mathbf{C}\mathbf{d}$$

$\Rightarrow \mathbf{C}, \mathbf{B}$ non-diagonal





# Finite Element Model – Contact Solver

## Equations and Inequalities

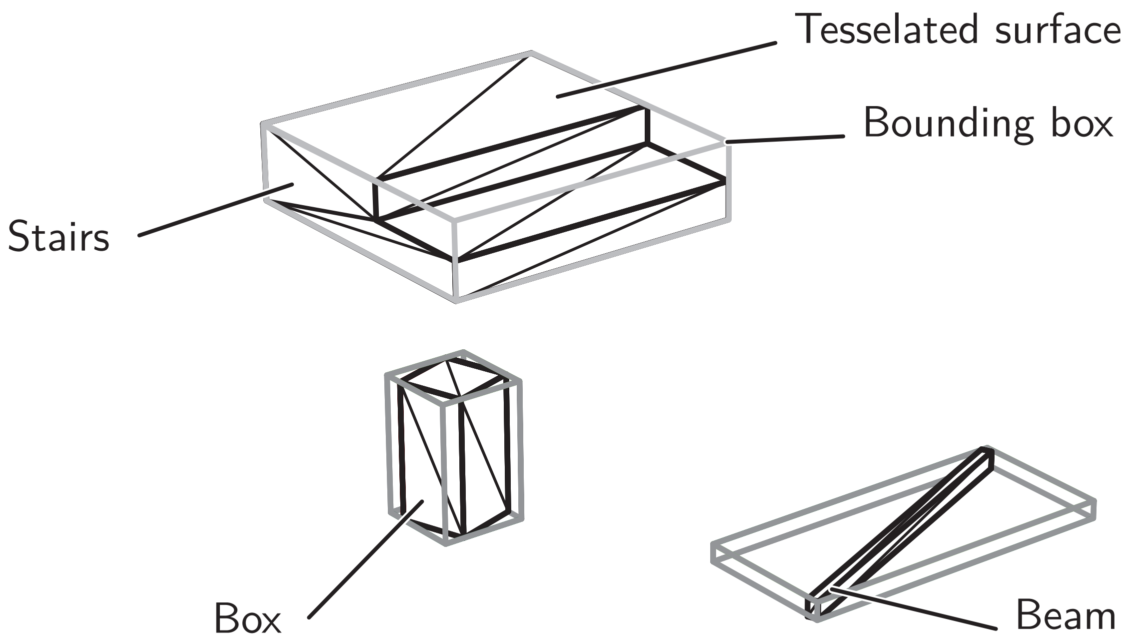
$$\mathbf{f} = \mathbf{B}\dot{\mathbf{d}} + \mathbf{C}\mathbf{d}$$
$$\lambda_{N,i} \geqslant 0 \ \wedge \ g_{N,i} \geqslant 0 \ \wedge \ \lambda_{N,i} g_{N,i} = 0 \quad \forall i \in I_c$$
$$\lambda_{N,i} - \mu \|\boldsymbol{\lambda}_{T,i}\| \geqslant 0 \ \wedge \ (\lambda_{N,i} - \mu \|\boldsymbol{\lambda}_{T,i}\|)\dot{\mathbf{g}}_{T,i} = 0 \quad \forall i \in I_c$$

## Numerical Solution

Reformulate complimentarity with proximal point functions:

$$\lambda_{N,i} = \text{prox}_{\mathscr{C}_N}(\lambda_{N,i} - r\dot{g}_{N,i}) \qquad \forall i \in I_c$$
$$\boldsymbol{\lambda}_{T,i} = \mathbf{prox}_{\mathscr{C}_T(\lambda_N)}(\boldsymbol{\lambda}_{T,i} - r\dot{\mathbf{g}}_{T,i}) \qquad \forall i \in I_c$$

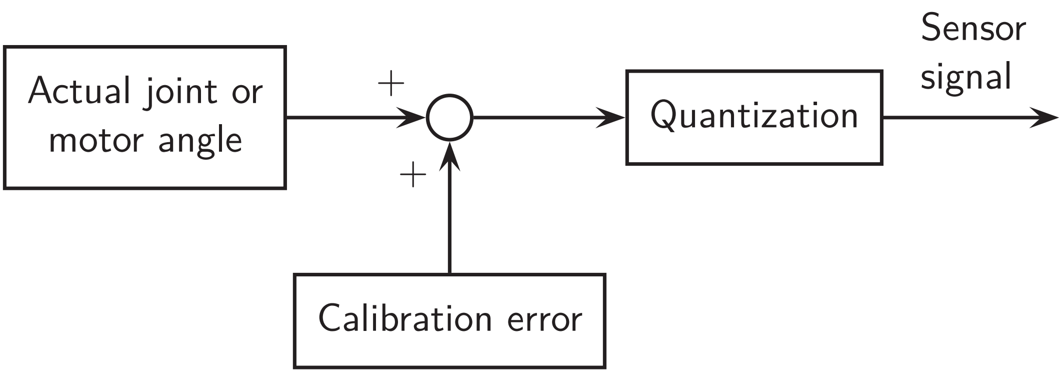
# Environment Model and Contact Determination



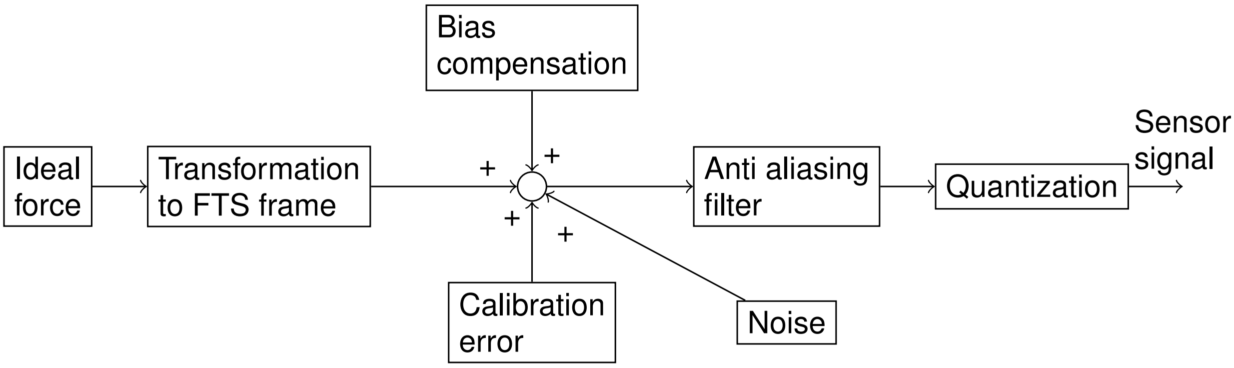





# Encoders



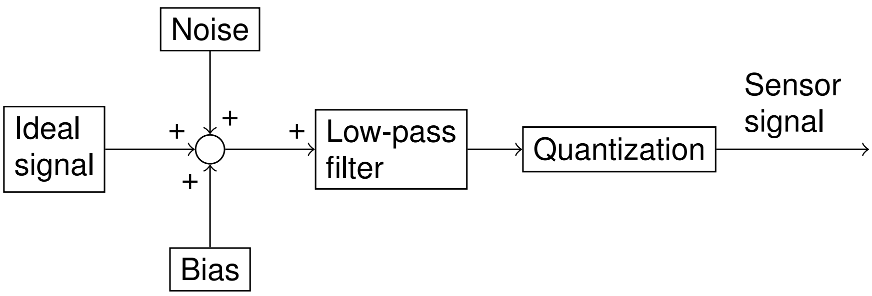
# Force/Torque Sensors





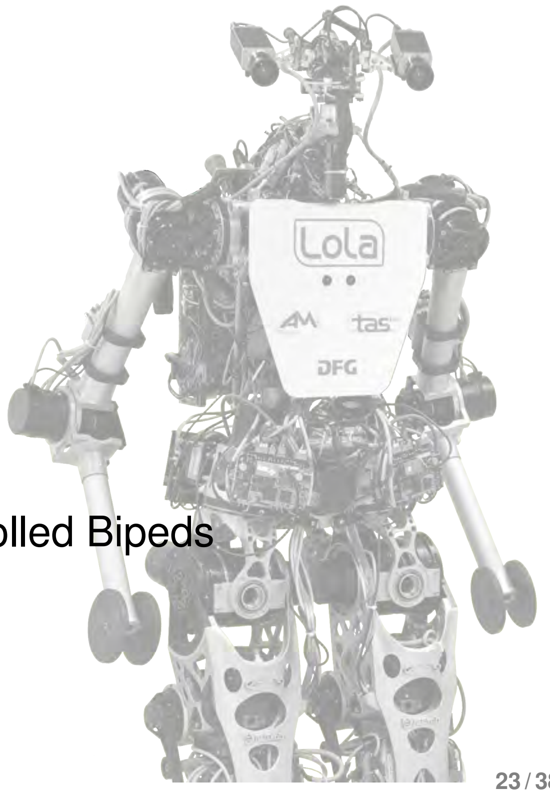



# Inertial Measurement Unit



# Outline








Introduction Model Components **Equations of Motion** Results Summary and Outlook

# Equations of Motion

Viscoelastic contact

Electric Motors

Multibody system

$$M\ddot{q} + h = W_\tau \tau + W_\lambda \lambda$$

$$L\dot{I} + RI + k_M \omega_{\text{rot}} = U$$

$$B\dot{d} + Kd = f$$

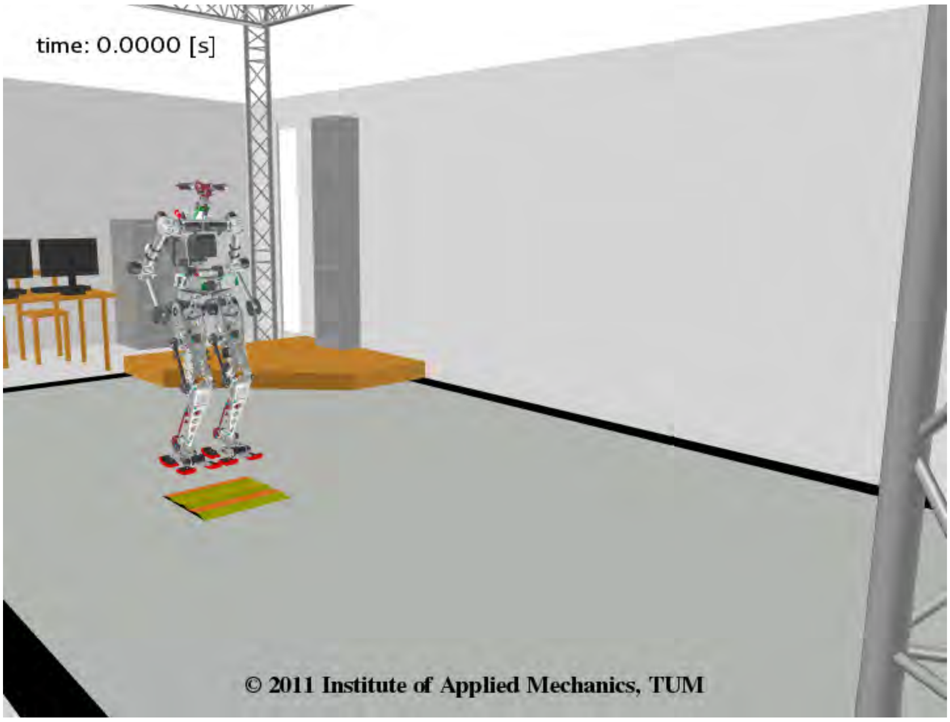
$$\lambda_{N,i} \geqslant 0 \;\wedge\; g_{N,i} \geqslant 0 \;\wedge\; \lambda_{N,i} g_{N,i} = 0 \;\forall i$$

$$\lambda_{N,i} - \mu \|\boldsymbol{\lambda}_{T,i}\| \geqslant 0 \;\wedge\; (\lambda_{N,i} - \mu \|\boldsymbol{\lambda}_{T,i}\|)\dot{\boldsymbol{g}}_{T,i} = 0 \;\forall i$$

Friction

Unilateral contact

# Simulation Example








# Reduced Order Model

### Why a Reduced Model?

- To reduce simulation time!
- Often, only global system behavior is relevant
- For good joint tracking control, error is small
- Limiting case of perfect joint control

# The Position Controlled System

## Multibody Model

- Torso degrees of freedom

$$\begin{pmatrix} M_{TT} & M_{TJ} \\ M_{JT} & M_{JJ} \end{pmatrix} \begin{pmatrix} \ddot{q}_T \\ \ddot{q}_J \end{pmatrix} + \begin{pmatrix} h_T \\ h_J \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ W_{\tau,J} \end{pmatrix} \tau + \begin{pmatrix} W_{\lambda,T} \\ W_{\lambda,J} \end{pmatrix} \lambda$$

- Joint angle degrees of freedom





# The Position Controlled System

## With High Gain Position Control

$$\tau \approx K \Delta q_J \qquad \text{with:} \quad K \gg 0$$

$$\Rightarrow \ \|\Delta q_J\| \approx 0 \qquad \text{or:} \quad q_J \approx q_{J,d}$$

$$\Rightarrow \ M_{TT}\ddot{q}_T + (M_{TJ}\ddot{q}_J + h_T) = W_{\lambda,T} W_{\lambda,T}$$

$$\Leftrightarrow \ M_{TT}\ddot{q}_T + h_T^{\star} = W_{\lambda,T}\lambda$$

# Dynamic Models of Lola

| | Reduced model | Full models | | | |
|---|---|---|---|---|---|
| Model component | Type 1 | Type 2 | Type 3 | Type 4 | Type 5 |
| Rigid body dynamics | yes | yes | yes | yes | yes |
| Gear elasticity | no | no | yes | no | yes |
| Drive dynamics | no | yes | yes | yes | yes |
| Contact model | SD | SD | SD | FEM | FEM |
| Mechanical DoFs | 6 | 30 | 46 | 30 | 46 |
| Contact layer DoFs | 48 | 48 | 48 | 864 | 864 |
| Electrical DoFs | 0 | 24 | 24 | 24 | 24 |
| No. of 1st order ODEs | 60 | 132 | 164 | 948 | 980 |

SD = Spring-Damper, FEM = Finite Element Model





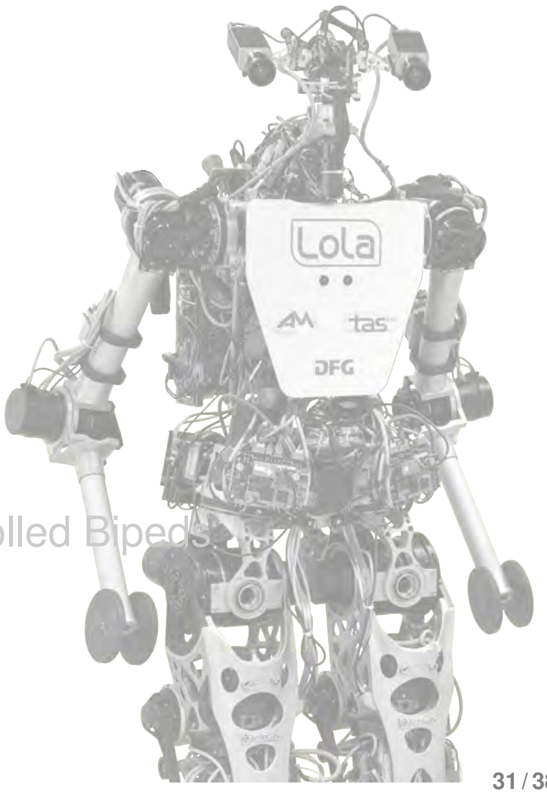
# Runtime Comparison

| Model type | Runtime/simulated time | Runtime/time step [s] | Time step [s] |
|---|---|---|---|
| Type 1 (reduced) | 0.64 (no I/O: 0.49) | $6.4 \times 10^{-4}$ | $10^{-3}$ |
| Type 2 (rigid) | 2.26 | $2.2 \times 10^{-4}$ | $10^{-4}$ |
| Type 3 (elastic) | 157.3 | $1.6 \times 10^{-4}$ | $10^{-5}$ |
| Type 4 (rigid, FEM) | 69.3 | $6.9 \times 10^{-3}$ | $10^{-4}$ |
| Type 5 (elastic, FEM) | 642.4 | $6.4 \times 10^{-3}$ | $10^{-5}$ |

Single threaded code, explicit Euler integration, on Intel Core i5-660 CPU (3.33 GHz)

# Outline

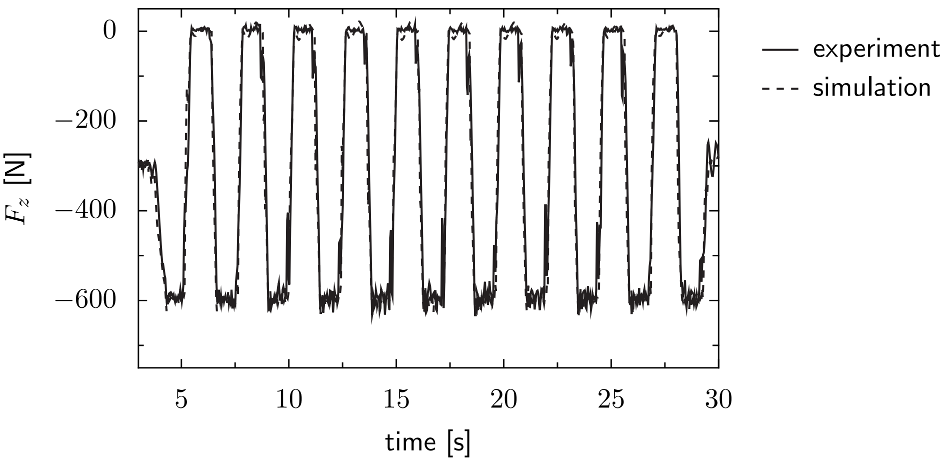




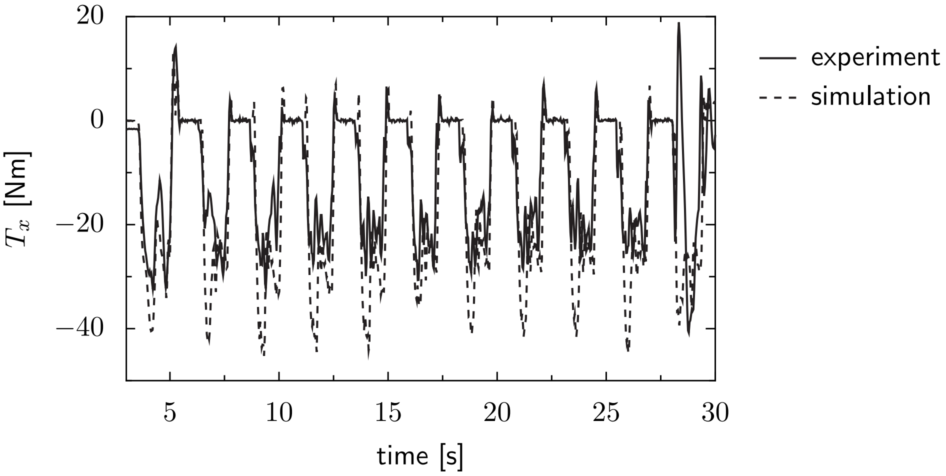




# Contact Force



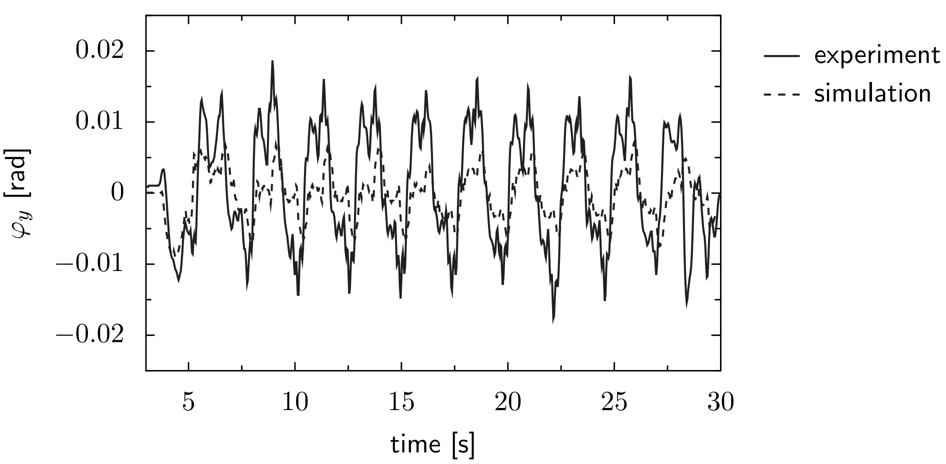
# Contact Torque



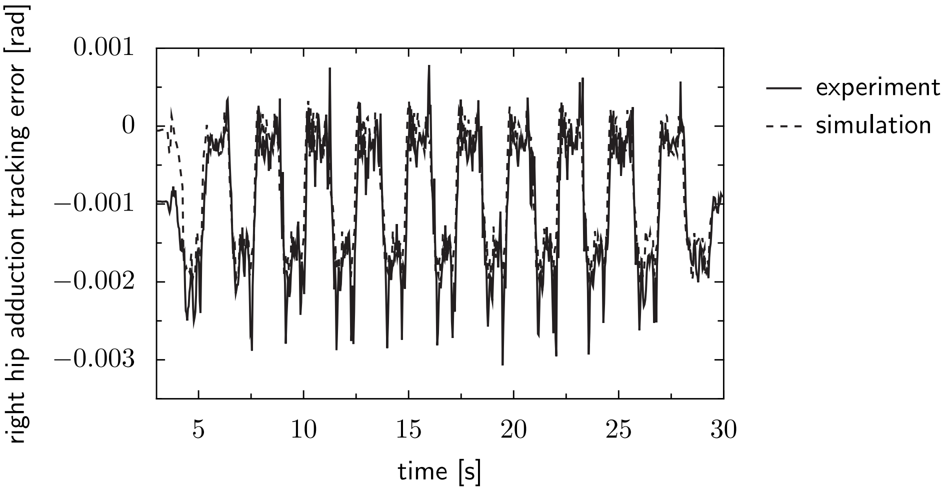





# Upper Body Inclination



# Joint Tracking Error

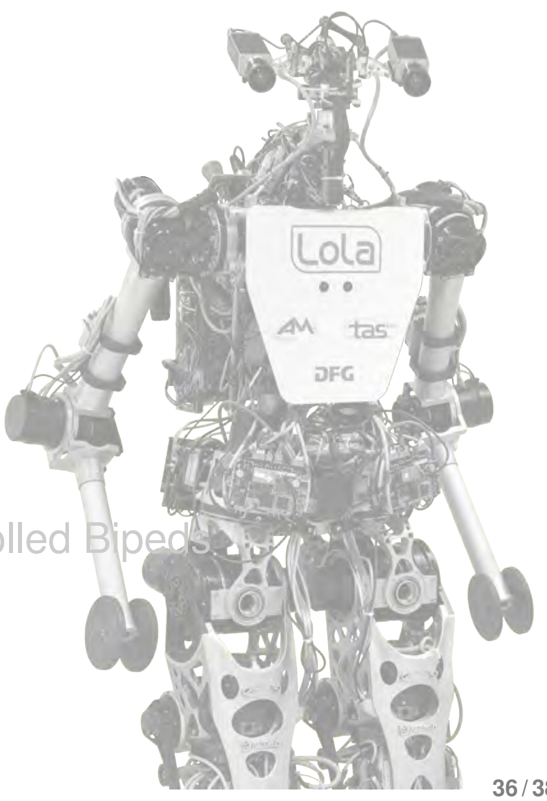







# Outline





# Summary and Outlook

**Summary**

- Modular simulation system
- Model components
- Comparison to measurements

**Outlook**

- Improve parameter identification
- Improve simulation speed



# Thank You!