Qubit transfer with high fidelity in 1D fermion spin chains with nearest-neighbour interaction based on new recurrence relations for Racah polynomials

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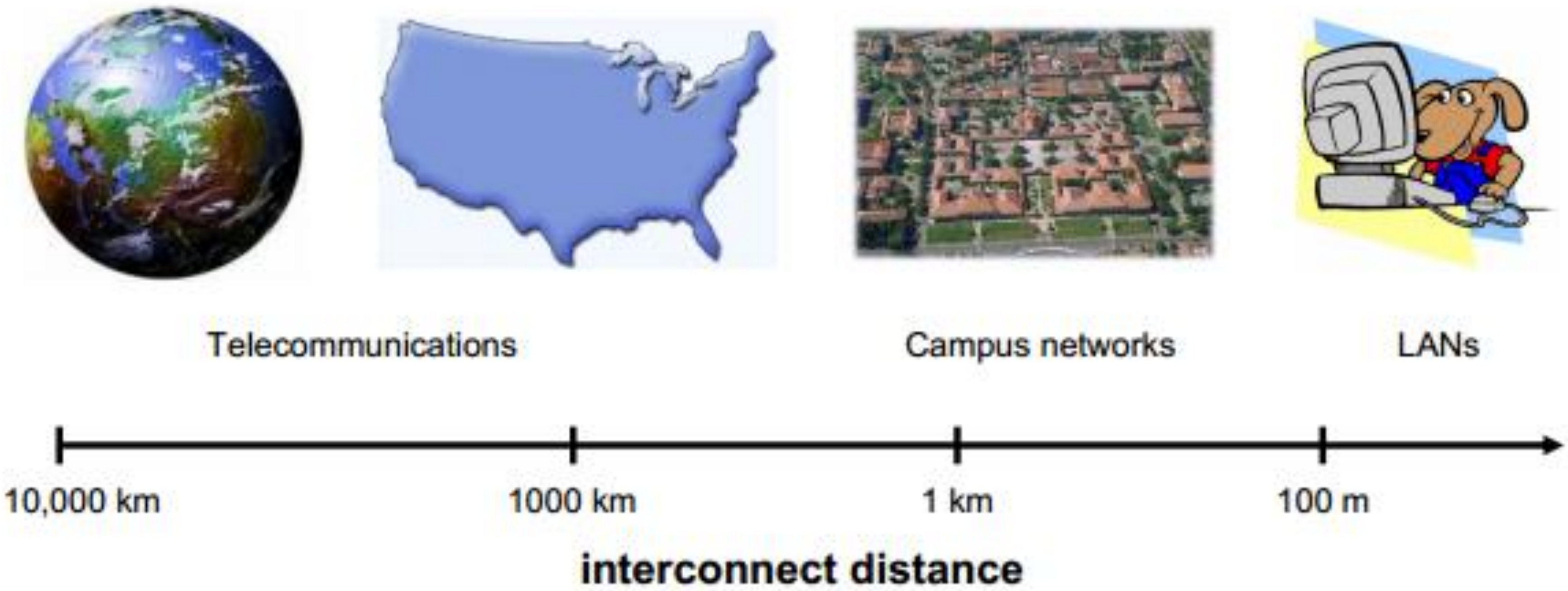
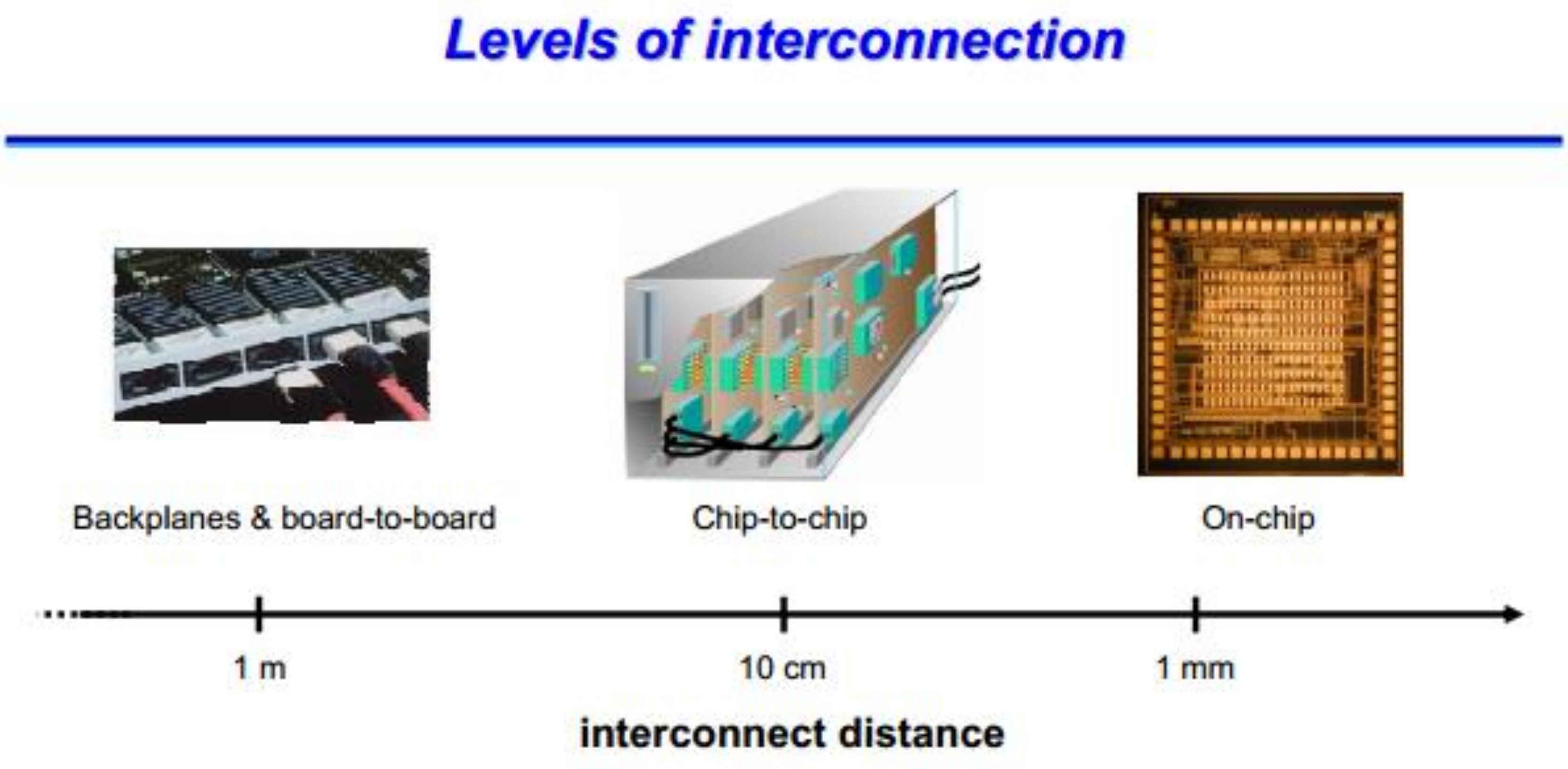
Levels of interconnection



Optics currently dominates for long distance interconnects
Increasingly, optics is used in local area network applications



Electrical signaling within computers is encountering severe limitations – what can help at these << 1mm length scales?

What is a quantum computer?

- A quantum computer is a machine that performs calculations based on the laws of quantum mechanics, which is the behavior of particles at the sub-atomic level.

Classical information

Quantum information

Stored as string of bits 0100110

Stored as quantum state of string of qubits $|\Psi\rangle$

Manipulation of (qu)bits (computation, dynamics)

Bit transformations (function computation)
All functions can be computed reversibly.

Unitary operations *U* (reversible)

Bit states can be copied.

Qubit states cannot be copied, except for *orthogonal* states

$$|\psi\rangle|0\rangle \underset{U}{\rightarrow} |\psi\rangle|\psi\rangle$$

$$\langle\psi|\phi\rangle^2 = \langle\psi|\phi\rangle$$

Transmission of (qu)bits (communication, dynamics)

Readout of (qu)bits (measurement)

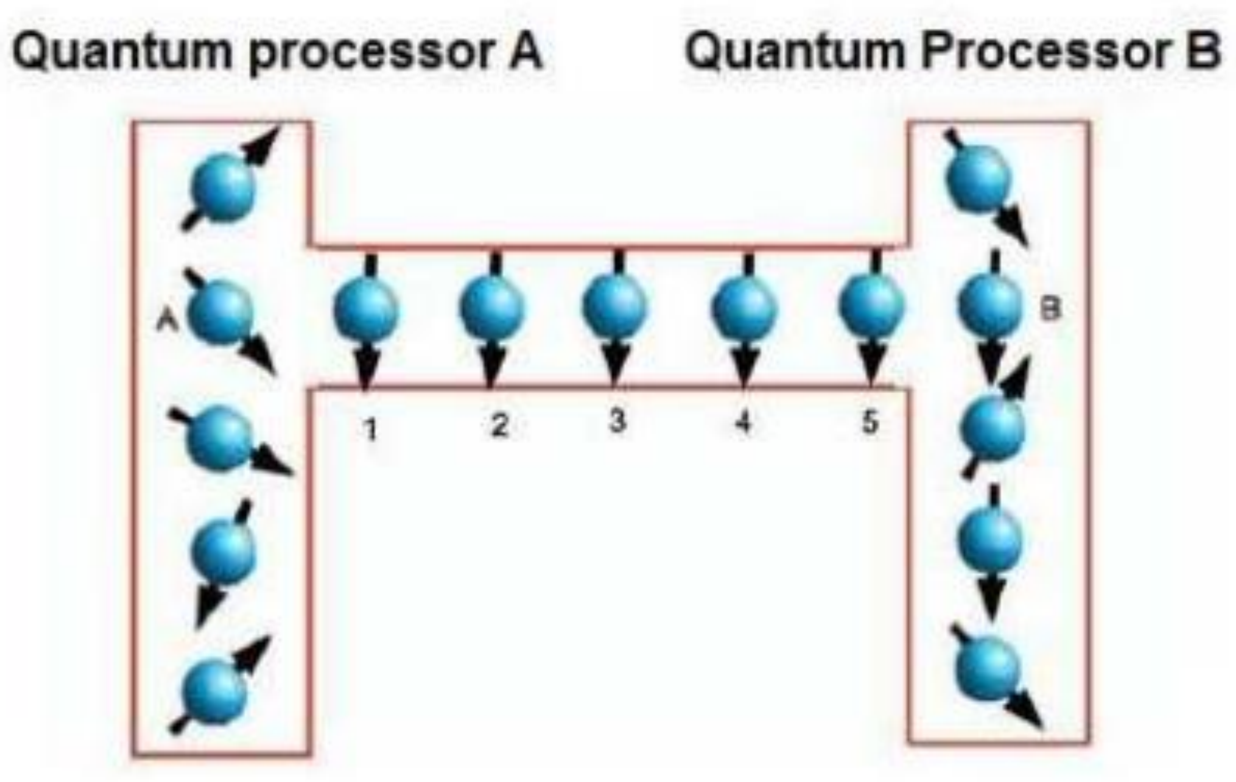
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$p_0 = |\alpha|^2 = |\langle 0|\psi\rangle|^2$$
$$p_1 = |\beta|^2 = |\langle 1|\psi\rangle|^2$$

Distinguishability of bit states

Quantum states are not distinguishable, except for *orthogonal* states

Quantum processors connected by spin chains

Realization is based on spin chain of the permanent fermions (spin-1/2 particles) with nearest-neighbour interaction: |0> - spin down, |1> - spin up.



Spin chains – recent development

VOLUME 91, NUMBER 20 PHYSICAL REVIEW LETTERS week ending 14 NOVEMBER 2003

Quantum Communication through an Unmodulated Spin Chain

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We propose a scheme for using an unmodulated and unmeasured spin chain as a channel for short distance quantum communications. The state to be transmitted is placed on one spin of the chain and received later on a distant spin with some fidelity. We first obtain simple expressions for the fidelity of quantum state transfer and the amount of entanglement sharable between any two sites of an arbitrary Heisenberg ferromagnet using our scheme. We then apply this to the realizable case of an open ended chain with nearest neighbor interactions. The fidelity of quantum state transfer is obtained as an inverse discrete cosine transform and as a Bessel function series. We find that in a reasonable time, a qubit can be directly transmitted with better than classical fidelity across the full length of chains of up to 80 spins. Moreover, our channel allows distillable entanglement to be shared over arbitrary distances.

DOI: 10.1103/PhysRevLett.91.207901 PACS numbers: 03.67.Hk, 05.50.+q, 32.80.Lg

721 times cited

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Spin chains – recent development

VOLUME 92, NUMBER 18 PHYSICAL REVIEW LETTERS week ending 7 MAY 2004

Perfect State Transfer in Quantum Spin Networks

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We propose a class of qubit networks that admit the perfect state transfer of any quantum state in a fixed period of time. Unlike many other schemes for quantum computation and communication, these

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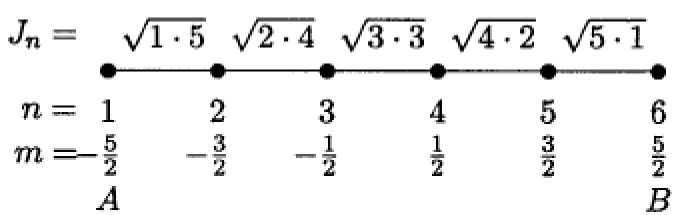

Spin chains – recent development

PRL 93, 230502 (2004) PHYSICAL REVIEW LETTERS week ending 3 DECEMBER 2004

Mirror Inversion of Quantum States in Linear Registers

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Transfer of data in linear quantum registers can be significantly simplified with preengineered but not dynamically controlled interqubit couplings. We show how to implement a mirror inversion of the state of the register in each excitation subspace with respect to the center of the register. Our construction is especially appealing as it requires no dynamical control over individual interqubit interactions. If, however, individual control of the interactions is available then the mirror inversion operation can be performed on any substring of qubits in the register. In this case, a sequence of mirror inversions can generate any permutation of a quantum state of the involved qubits.

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161 times cited

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Spin chains with nearest-neighbor interaction

Hamiltonian of spin chain of $(N+1)$ *electrons coupled via the nearest-neighbour interaction is the following*

Hamiltonian:

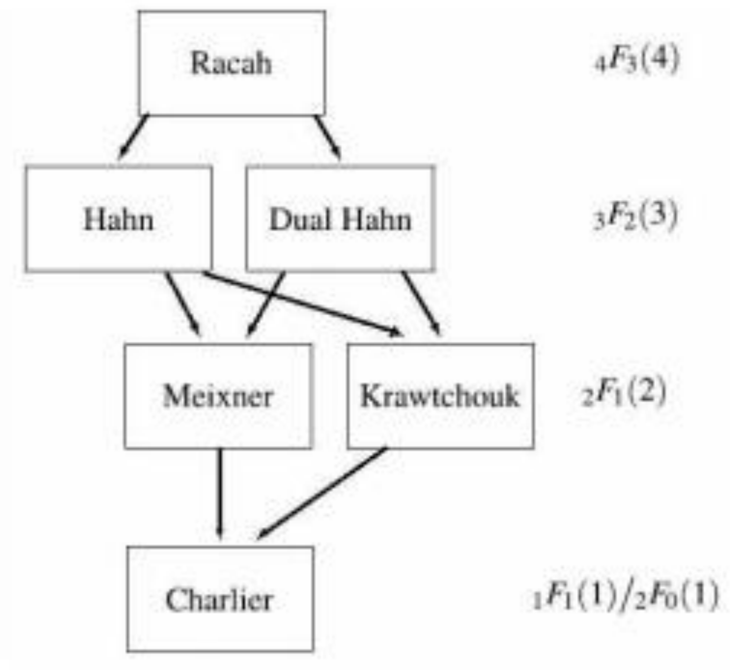
$$\hat{H} = \frac{1}{2}\sum_{k=0}^{N-1} J_k \left(\sigma_k^x \cdot \sigma_{k+1}^x + \sigma_{k+1}^y \cdot \sigma_k^y\right) + \frac{1}{2}\sum_{k=0}^{N} h_k \left(\sigma_k^z + 1\right)$$

J_k - coupling strength, h_k - Zeeman energy.

Pauli matrices:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Askey scheme of the orthogonal polynomials



- R. Koekoek, P.A. Lesky, R. Swarttouw, Hypergeometric Orthogonal Polynomials and Their q-Analogues, Springer (2010)

Spin chains – evolution in time

- Unitary time evolution operator: $\boldsymbol{U(t) = exp(-it\hat{H})}$
- "State sender" located at the site s, "State receiver" located at the site r
- When t=0, the system located at the sites s
- Transition amplitude (or correlation function)

$f_{r,s}(t) = \langle r|\mathcal{U}(t)|s\rangle =$ 

- Compute explicit expression $f_{r.s}(t)$

$$f_{r,s}(t) = \langle \sum_{k=0}^{N} U_{rk}\varphi_k | \exp(-it\hat{H}) \sum_{j=0}^{N} U_{sj}\varphi_j \rangle = \sum_{j=0}^{N} U_{rj} U_{sj} e^{-it\epsilon_j}$$

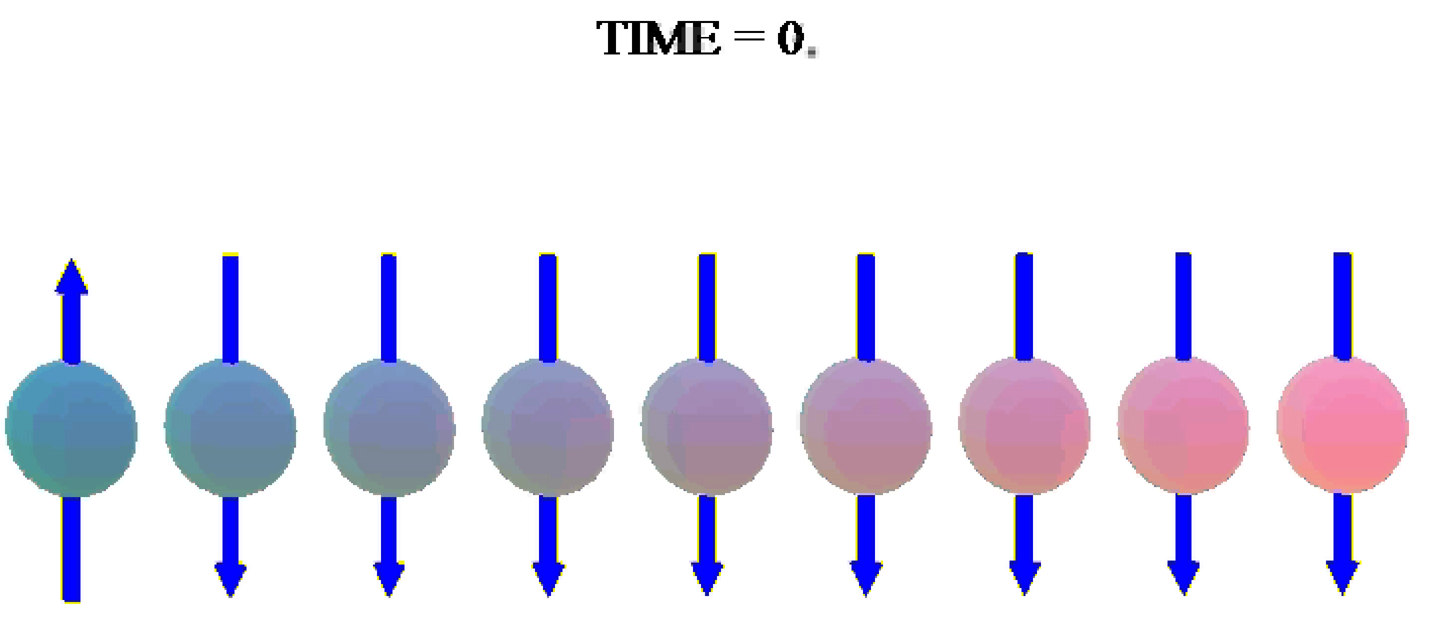
Spin chains – simplest example of analytical solution

▷ $\boxed{h_k = 0 \text{ and } J_k = 1}$.

$$\epsilon_j = 2\cos\left(\frac{(j+1)\pi}{N+2}\right) \qquad U_{ij} = \sqrt{\frac{2}{N+2}}\sin\left(\frac{(i+1)(j+1)\pi}{N+2}\right).$$

$$f_{r,s}(t) = \sum_{j=0}^{N} \frac{2}{N+2} \sin\left(\frac{(r+1)(j+1)\pi}{N+2}\right) \sin\left(\frac{(s+1)(j+1)\pi}{N+2}\right) \times \exp\left(-2it\cos\left(\frac{(j+1)\pi}{N+2}\right)\right)$$

Simulation



Spin chains – another example – Krawtchouk polynomials

▷ $h_k = N/2 \text{ and } J_k = \frac{1}{2}\sqrt{(k+1)(N-k)}$.

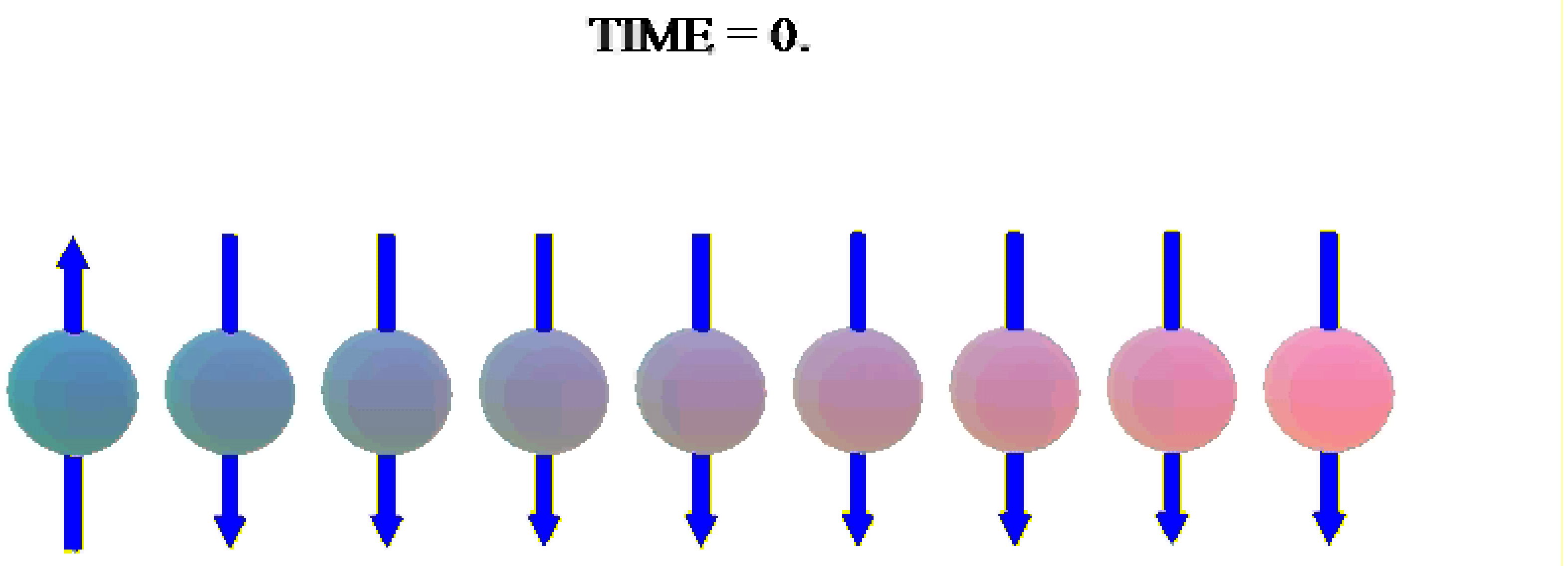
$\epsilon_j = j \; (j = 0, 1, \ldots, N)$ $U_{ij} = \tilde{K}_i(j) \sim {}_2F_1\left({-i, -j \atop -N}; \frac{1}{2}\right)$

$$f_{r,s}(t) = \sqrt{\binom{N}{r}\binom{N}{s}}\left(\sqrt{p(1-p)}\right)^{r+s}(1-z)^{r+s}(1-p+pz)^{N-r-s}$$
$$\times \; {}_2F_1\left({-r, -s \atop -N}; \frac{-z}{p(1-p)(1-z)^2}\right).$$

$$f_{N,0}(t) = \left(\sqrt{p(1-p)}\right)^N (1-e^{-it})^N$$

$f_{N,0}(\pi) = 1$ $p = 1/2$.

Simulation



Spin chains – another example – q-Krawtchouk polynomials

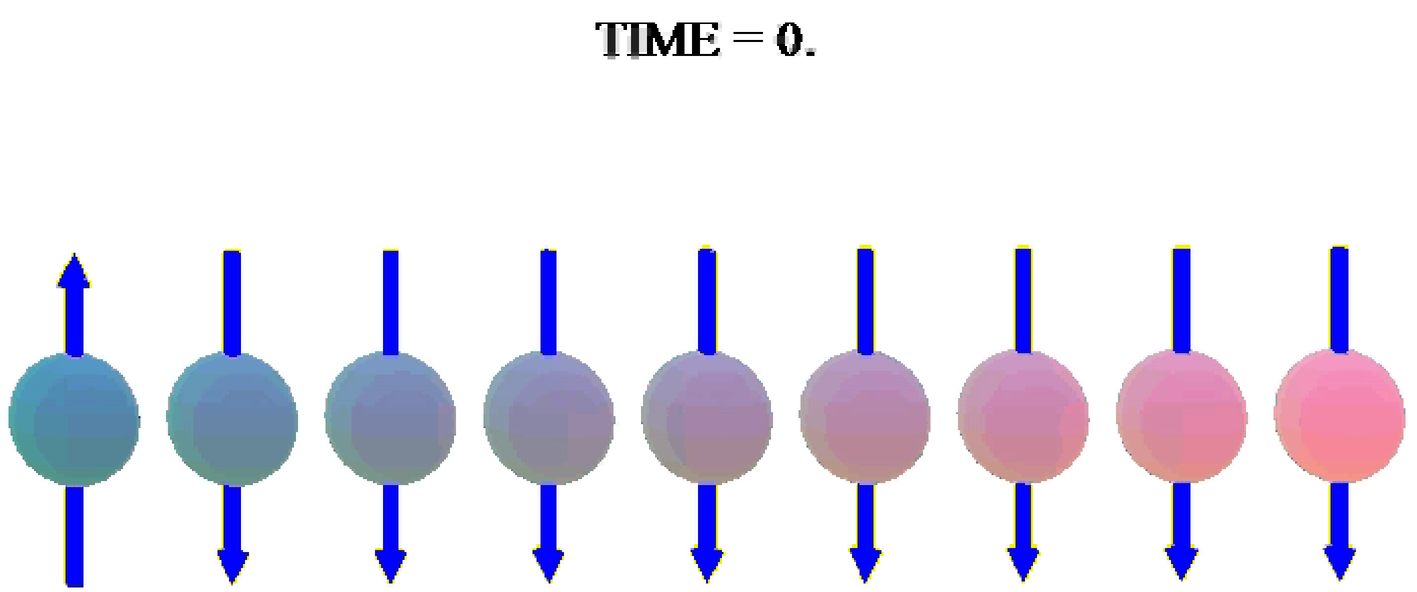
Jacobi matrix elements

$$J_n = -\frac{A_n}{1-q}\sqrt{\frac{d_{n+1}}{d_n}}, \quad h_n = -\frac{A_n + C_n}{1-q},$$

$$U_{jk} = \tilde{K}_j(q^{-k}) \qquad UU^T = U^T U = I.$$

$$\boxed{\epsilon_j = -[-j] = -\frac{1-q^{-j}}{1-q} = q^{-1} + q^{-2} + \cdots + q^{-j}.}$$

Simulation



Hamiltonian of spin chain (N+1) electrons

$$\hat{H} = \sum_{k=0}^{N-1} J_k (a_k^+ a_{k+1} + a_{k+1}^+ a_k)$$

Pair of new recurrence relations for Racah polynomials

$$\left[\frac{(x+\alpha+2)(x+\gamma+\delta+1)}{2x+\gamma+\delta+2} e^{\partial_x} - \frac{(x+1)(x-\alpha+\gamma+\delta)}{2x+\gamma+\delta+2}\right] R_n(\lambda(x);\alpha+1,\beta-1,\gamma,\delta)$$

$$= (\alpha+1) R_n(\lambda(x+1);\alpha,\beta,\gamma,\delta-1),$$

$$\left[\frac{(\beta-\gamma-x-1)(x+\delta)}{2x+\gamma+\delta+1} + \frac{(x+\gamma+1)(x+\beta+\delta)}{2x+\gamma+\delta+1} e^{\partial_x}\right] R_n(\lambda(x);\alpha,\beta,\gamma,\delta-1)$$

$$= \frac{(n+\alpha+1)(n+\beta)}{\alpha+1} R_n(\lambda(x+1);\alpha+1,\beta-1,\gamma,\delta).$$

Racah polynomials

$$R_n(\lambda(x);\alpha,\beta,\gamma,\delta)$$

$$= {}_4F_3\left(\begin{matrix} -n, n+\alpha+\beta+1, -x, x+\gamma+\delta+1 \\ \alpha+1, \beta+\delta+1, \gamma+1 \end{matrix}; 1\right),$$

$$\lambda(x) = x(x+\gamma+\delta+1), \qquad n = 0, 1, 2, \ldots, m$$

Coupling strength

$$J_k = \begin{cases} \sqrt{(k+1)(m-k)f(\alpha,\beta,\delta)}; & k - odd \\ \sqrt{(k+2\alpha+2)(m-k+2\beta)g(\delta)}; & k - even \end{cases}$$

$$f(\alpha,\beta,\delta) = \frac{(k-2\alpha+2\delta-m)(k+2\beta+2\delta-1)}{(2k+2\delta-m-1)(2k+2\delta-m+1)}$$

$$g(\delta) = \frac{(k-m+2\delta-1)(k+2\delta)}{(2k+2\delta-m-1)(2k+2\delta-m+1)}$$

During computations, it is necessary to take into account satisfaction of the following three cases, under which the Racah polynomials hold finite-discrete orthogonality relation

$$\frac{(-\beta)_m(\gamma+\delta+2)_m}{(-\beta+\gamma+1)_m(\delta+1)_m}, \qquad \text{if } \alpha+1=-m,$$

$$\frac{(\alpha+\delta)_m(\gamma+\delta+2)_m}{(-\alpha+\gamma+\delta+1)_m(\delta+1)_m}, \qquad \text{if } \beta+\delta+1=-m,$$

$$\frac{(\alpha+\beta+2)_m(-\delta)_m}{(\alpha-\delta+1)_m(\beta+1)_m}, \qquad \text{if } \gamma+1=-m.$$

Transition from one end of the chain ($s = 0$) to the final end of the chain ($r = N = 2m + 1$), $t \equiv T = \pi/2$;

Time evolution correlation function:

$$f_{N,0}(\pi/2) = \sqrt{\frac{(\alpha - \delta + 2)_m (\alpha + \delta + 1)_m}{(\delta)_m (1 - \delta)_m}}$$

Conclusion

- Time-dependent correlation function becomes equal to 1, is called as perfect state transfer. One can easily check that expression of the correlation function, obtained in this work, never becomes equal to 1.
- Selection of nearest-neighbour coupling parameters J_k, corresponding to pair of the recurrence relations for the Racah polynomials, one can assess only the case of the qubit transfer with the high fidelity. This statement is very important, because, it allows observing the law of the breakdown of the perfect qubit transfer under the certain complication of the nearest-neighbour interaction parameter.

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Thank you for attention!